ESTIMATION OF *G*(0) BASED ON THE SIGHTING SURVEY DATA AND COVARIATES INFORMATION OF JARPA

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ABSTRACT

The estimation model of $g(0)$ was developed using the data of whales sighting survey and surfacing information of whale without the independent observer experiment. Moving coordinate system was defined as the position of observer was fixed at origin. The surfacing probability in a unit time Δs and surfacing detection probability function Q were also defined. The function of Q is considered many covariates such as sighting survey condition, group size of whale and so on. $g(0)$ was estimated by maximizing the likelihood of Q on the data of primary observation information of whales group. Results of simulation examination suggest that unbiased estimators were obtained from this method. g(0) of JARPA sighting survey on the area IV was estimated by the proposed method. The g(0) were estimated as 0.5 - 0.7 and were significantly different to one.

Key words: $g(0)$, non-IO, surfacing probability, Covariates

Line transect methods are widely used to estimate density and population of cetaceans. The traditional line transect methods have strong assumption that the probability of objects detection on the track line, $g(0)$, is 100%. However, this assumption does not match in whale survey (Doi et al., 1983; Hiby, 1986; Øien, 1990; Schweder et al., 1991b; Cooke, 1994). If the whales on the track line are not detected, the number of whale is underestimated with traditional line transect method.

Therefore to estimate $g(0)$ is necessary to obtain unbiased number of whale. The independent observer (IO) experiment and the estimation model of $g(0)$ are widely used. On the other hand, many sighting surveys have not performed IO because of the limitation of facilities or observers on the survey vessel.

A few studies have been reported on the $g(0)$ estimation without IO experiments. Doi et al. (1983) estimated $g(0) = 0.699$ for Antarctic minke whale by using Monte-Carlo simulation. Kasamatsu and Joyce (1995) estimated $g(0)$ of odontocete by using simulation method. Okamura (2003) examined the accuracy and bias of $g(0)$ estimated from the part of IO model which can apply to the survey data of traditional line transect from generated pseudo-observation data by using simulation. He reported that true $g(0)$ can be estimated from the model added average of surfacing interval and term of group size. These studies assumed that the conditions during the surveys were constant.

However, some studies suggested that survey conditions affect the observation efficiency of the survey. Doi et al. (1983) indicated that effort of observation differs by the angle between track line and the target. Barlow (2001) presented that perpendicular distance was affected by group size, Beaufort scale, cue, vessel, year and survey area. Murase et al. (2004) reported that detection distance was shorter when Beaufort scale was larger and cue was body than brow. The estimation of *g*(0) added these factors is not studied enough.

The population estimation of the Antarcic minke whale on the area IV by using traditional line transect method from JARPA data had a twice discrepancy between the estimated population of 97/98 and 99/00. One of the reason of this discrepancy has been thought to be the difference of g(0) or survey conditions.

In the current study, a *g*(0) estimation model without independent observer program was proposed to estimate population density of cetacean from the data of cetacean sighting survey and covariates of survey conditions such as Beaufort, visibility and so on. The impact of such covariates was also examined by using pseudo-observation data generated from Monte-Carlo simulation. Furthermore, the model was applied for sighting data of JARPA on area IV and estimated g(0) and the population of Antarctic minke whale.

METHODS

g(0) estimation model using non-IO data

Survey vessel was fixed at the origin of moving coordinate system and was headed to the Y-axis. This X-Y coordinate indicates the relative position of whale groups (including whale group which the size of it equals to one) to the vessel. Assuming the vessel speed *v* is enough faster than whales one, a position of whale group is described as $(x, y - v\tau)$, where (x, y) is whale position on time *t*, and τ is elapsed time from time *t* (Fig. 1). Whale behavior was categorized into two; surfacing and diving. The probability of surfacing or diving were assumed to be independent to the vessel position and time. A surfacing whale has a possibility to be observed by the observers on the vessel, as far as $y < y_{\text{limit}}$, where y_{limit} is a large vertical distance enough not to be able to detect.

Expectation surfacing probability Δs was defined as the average probability that a whale surface in the interval of $\Delta\tau$ from the time that the whale pass a certain point (x, y). Assuming the behavior of whale is independent on time or relative position from the vessel, Δs is a constant. The derivation of Δs was described in appendix.

We defined the surfacing detection probability subject to the whale surfaces at (x, y), $Q(x, y, \mathbf{H} \mid \mathbf{a})$, where $\mathbf{H} = \{\eta_1, \cdots, \eta_m\}$ is a vector of covariates and assumed to be constant for a whale group which enter the area $y < y_{\text{limit}}$, $\mathbf{a} = \{a_1, \cdots, a_l\}$ is a vector of free parameters. Q is a function describing probability, thus $0 \le Q \le 1$.

For making the discrete model on Y axis, the distance y_{limit} is divided in *c* bands evenly. The width of the band is calculated as y_{limit}/c and the speed of vessel was set at *v*. Then, the time to pass a band $\Delta\tau$ is calculated as follows;

$$\Delta\tau = \frac{y_{\text{limit}}}{c \cdot v} \tag{1}$$

The interval of band *i* was defined as

$$(i-1)\Delta\tau v \le y \le i\Delta\tau v, \quad i = 1, \cdots, c. \tag{2}$$

where *i* is interval number, *i*=1 is the nearest interval to vessel on the origin (Fig. 2).

Here, we redefined expected surfacing detection probability that a whale on the line with the perpendicular distance *x*, was detected in the band of *i* if it surfaced on that band; $Q(x, (i-0.5)\Delta\tau v, \mathbf{H} \mid \mathbf{a})$, and abbreviated to $Q_i(x, \mathbf{H} \mid \mathbf{a})$.

Furthermore probability that observer primary detected whale on the line with perpendicular distance x or the primary detection probability $q_i(x, \mathbf{H} \mid \Delta s, \mathbf{a})$ was defined as follows. A whale group on (x, y_{limit}) achieves to $(x, y_{\text{limit}} - \Delta \tau v)$ after time interval $\Delta \tau$. It passed band i=c and expected surfacing probability is Δs from the definition of it. Therefore, detection probability on the band c is given by

$$q_c(x, \mathbf{H} \mid \Delta s, \mathbf{a}) = \Delta s \cdot Q_c(x, \mathbf{H} \mid \mathbf{a}). \tag{3}$$

q_{c-1} is calculated as follows;

$$q_{c-1}(x, \mathbf{H} \mid \Delta s, \mathbf{a}) = (1 - q_c) \cdot \Delta s \cdot Q_{c-1}(x, \mathbf{H} \mid \mathbf{a}) \tag{4}$$

Similarly, q_i is obtained as recurrence equation,

$$q_i(x, \mathbf{H} \mid \Delta s, \mathbf{a}) = \left(1 - \sum_{k=i+1}^{c} q_k\right) \cdot \Delta s \cdot Q_i(x, \mathbf{H} \mid \mathbf{a}). \tag{5}$$

The $g(x)$ is the detection probability from y_{limit} to y=0 and equals $\sum_{i=1}^{c} q_i(x)$. That is

$$g(x, \mathbf{H} \mid \Delta s, \mathbf{a}) = \sum_{i=1}^{c} q_i(x, \mathbf{H} \mid \Delta s, \mathbf{a}). \tag{6}$$

Note $q_{i>c} = 0$, as equation (5) is derived from Q and Δs, $g(x)$ can be calculated from them.

In order to obtain $g(x)$ from Equation (5), the parameters $\mathbf{a}$ were estimated by maximum likelihood method using detection probability density distribution, q_{pdf}, obtained from q under particular $\mathbf{H}$ as follows;

$$q_{\text{pdf},i_j}(x_j, \mathbf{H}_j \mid \Delta s_j, \mathbf{a}) = \frac{c}{y_{\text{limit}}} \cdot \frac{1}{w(\mathbf{H}_j \mid \Delta s_j, \mathbf{a})} \cdot q_{i_j}(x_j, \mathbf{H}_j \mid \Delta s_j, \mathbf{a}), \tag{7}$$

where i_j is detected interval of whale group j, x_j, $\mathbf{H}_j$ and Δs_j is detected perpendicular distance, covariates and expected surfacing probability of whale group j respectively and w is effective strip width under $\mathbf{H}_j$ and Δs_j that is calculated from

$$w(\mathbf{H} \mid \Delta s, \mathbf{a}) = \int_{-\infty}^{\infty} g(x, \mathbf{H} \mid \Delta s, \mathbf{a}) dx \tag{8}$$

In equation (7), $\dfrac{c}{y_{\text{limit}}} \cdot \dfrac{1}{w(\mathbf{H}_j \mid \Delta s, \mathbf{a})}$ was the value to normalize q. Log-likelihood function for detection was calculated by

$$\begin{aligned} LL(\mathbf{a} \mid \Delta s, \mathbf{H}, \mathbf{x}, \mathbf{i}) &= \sum_{j=1}^{n} \ln\left\{q_{\text{pdf},i_j}(x_j, \mathbf{H}_j \mid \Delta s_j, \mathbf{a})\right\} \\ &= \sum_{j=1}^{n} \left[\ln\left\{q_{i_j}(x_j, \mathbf{H}_j \mid \Delta s_j, \mathbf{a})\right\} - \ln\left\{w(\mathbf{H}_j \mid \Delta s_j, \mathbf{a})\right\} + \ln \frac{c}{y_{\text{limit}}}\right] \end{aligned} \tag{9}$$

$g(0)$ was estimated from maximum likelihood estimators $\mathbf{a}^*$ that was calculated from the equation (9), as follows;

$$g(x = 0, \mathbf{H} \mid \Delta s, \mathbf{a}^*) = \sum_{i=1}^{c} q_i(x = 0, \mathbf{H} \mid \Delta s, \mathbf{a}^*) \tag{10}$$

Furthermore group density D^*, individual density D^*_{ind} around the track line and number of whale was derived by

$$D^* = \sum_{j=1}^{n} \frac{1}{w_j^*(\mathbf{H}_j \mid \Delta s_j, \mathbf{a}^*) \cdot L}, \tag{11}$$

$$D^*_{\text{ind}} = \sum_{j=1}^{n} \frac{s_j}{w^*_j(\mathbf{H}_j \mid \Delta s_j, \mathbf{a}^*) \cdot L} \tag{12}$$

$$N^* = A \cdot D^*_{\text{ind}} \tag{13}$$

respectively, where s_j is group size of j, and A is survey area. Confidence interval of them was estimated by Bootstrap method.

Simulation

The simulation tests were conducted in order to examine that this procedure estimate properly, and to examine the effect of covariates using pseudo-observation data.

The data was generated as follows. A virtual sea was created with 15 nmile width (the range of X-axis is –7.5 to 7.5 nmile), and with 200 nmile length (the range of Y-axis is 0 to 200 nmile) on computer. The sea length was divided in c=1000 bands. The vessel fixed at the origin and was headed to the Y-axis. The virtual sea moved backward along Y-axis with speed v = 11.5kt. The vessel searched on the sea surface (the range of X-axis was -7.5 to 7.5 nmile and Y-axis is 0 to y_{limit} =7.5 nmile) and detected a surfacing whale with surfacing detection probability as follows;

$$Q(dist, ang \mid \mathbf{H}, \mathbf{a}) = \exp\left[-\left(\begin{array}{l} a_0 dist \cdot \{1 + a_2 \sin^2(|ang|)\} \cdot \\ \qquad \left(\dfrac{2}{1 + e^{a_3 (s-1)}} \right) \cdot (1 + a_4 \eta_1) \cdot (1 + a_5 \eta_2) \end{array} \right)^{a_1} \right] \tag{14}$$

$$a_0 = 3.5, a_1 = 0.8, a_2 = 0.6, a_3 = 0.1, a_4 = 0.6, a_5 = 0.7$$

where $dist$ and ang are the distance and angle from the vessel to the whale respectively and two covariates $\eta_1 = \{0,1,2,3,4\}$ and $\eta_2 = \{0,1\}$ were defined in the vector $\mathbf{H}$. The whale groups of size s=1,2,…,5 were distributed randomly with density $D(s)$ (0.05, 0.027, 0.013, 0.007 and 0.003 ind./nmile^2 respectively) and repeated surfacing and diving at the same point on the virtual sea according to surfacing interval *p.d.f.* as fallows;

$$f(\tau, s) = \frac{1}{\lambda(s)} \exp\left[-\frac{\tau}{\lambda(s)} \right] \tag{15}$$

$$\lambda(s) = 166.92 \cdot e^{-0.25s}$$

In this model, the parameter function $\lambda(s)$ were adjusted as the average surfacing interval of a whale group with s=1 is 130 sec. If whale surfaced, the next surfacing timing was selected randomly according to $f(\tau, s)$. Each whale group j carried covariates factors $\mathbf{H}_j$, hence one group has two types of covariates $\eta_1 = \{0,1,2,3,4\}$ and $\eta_2 = \{0,1\}$ and the $size = \{1,2,3,4,5\}$. Therefore 50 types of different group were appeared on the virtual sea. If whale passed across the X-axis, the whale was removed once and a new whale was generated at (x, 200) where x was selected randomly. When observer detected a whale group, the detection distance *dist*, angle *ang*, group size s and covariates $\mathbf{H}_j$ was recorded. The observation was terminated when the number of detection reached to n=400, and survey length L was recorded. Same survey was repeated 30 times.

From the virtual sighting data obtained above, $g(0)$ and density was estimated using $g(0)$ estimation model under the true Q described equation (14) and compared with true $g(0)$ and density. The true $g(0)$ was calculated from true Q using equation (10). True individual density was calculated from $D_{ind} = \sum_{s=1}^{\max} sD(s)$. Individual density D^*_{ind} was calculated using estimated $g^*(0)$, effective strip width w^* and length of track line L. Furthermore, the average, coefficient of variance and quartile (25%$^{\text{ile}}$, 50%$^{\text{ile}}$, 75%$^{\text{ile}}$) was calculated by estimated values from 30 sighting data respectively.

For examining the misspecification of the model of Q, another Q model was also used which leave out covariate terms from equation (14), as follows;

$$Q(dist, ang \mid \mathbf{H}, \mathbf{a}) = \exp\left[-\left(a_0 dist \cdot \{1 + a_2 \sin^2(|ang|)\} \cdot \left(\frac{2}{1+e^{a_3(s-1)}}\right)\right)^{a_1}\right] \quad (16)$$

The result was compared using the quartile of D^*_{ind} and $g^*(0)$ with true value.

Application on the JARPA data

The SSV sighting data, survey condition data and survey effort data of JARPA from 95/96 to 01/02 only in area IV were used for the application.

For the survey condition data which is the additional data of the sighting data, detection distance, detection angle, size of whale group, cue of the detection , Beaufort scale, wind speed, and pack ice coverage were used for the full model as the covariates and significant covariates were selected by using GLM by year. Length of track line, area and ship velocity was also used following Hakamada et al. (2001). A total of 60 individual and 657 surfacing data was obtained from the chasing experiment on 90/91 to 96/97 and was used for the application.

The confidence intervals of the estimators were obtained by using bootstrap method. Because of the small number of replication (30), the errors of the estimators were assumed to be log-normal distribution and the 95% confidence interval were estimated from the distribution function fitted to the 30 estimators obtained from the bootstrap samples.

RESULT

From the result of simulation examinations, unbiased $g(0)$s were estimated on all combination of size and other covariates under true Q model was given (Fig. 4). The CVs are 0.30 - 9.18%. True $g(0)$s were included between 25%^{ile} and 75%^{ile} of the estimated $g(0)$ under all combination.

$g(0)$s from limited Q model were estimated to nearly similar value in the same size in spite of the variation of covariates (Fig. 5). 48 of 50 true $g(0)$s were out of the range of quartiles of estimated $g(0)$.

From the analysis of the chasing experiment of JARPA on 90/91 to 96/97, the average surfacing interval was obtained as Table 1. Survey length of each year and stratum used in this study was shown in Fig. 6. Average group size in each year and stratum was shown in Fig. 7. Fig. 8 shows the average Beaufort index.

From the GLM analysis, angle, size and cue were selected for all four years and Beaufort, wind velocity or pack ice coverage were selected for some years as the significant covariates.

The estimated $g(0)$ value was shown in Fig. 9. In 95/96, $g(0)$ was estimated about 0.5 and was lower than the other years. In 97/98 to 01/02, $g(0)$ were estimated quite same value as 0.7. The differences between estimated $g(0)$ and 1 were significant for all years.

Fig. 10 and 11 show the estimated group density and individual density. The estimated number of group and number of individuals are shown in Fig. 12 and 13.

The total population estimated from the current study was about 40% higher than the traditional estimators (Hakamada et al. 2001)(Fig. 14). Especially in 1995, the population was estimated twice comparing the traditional one.

DISCUSSION

As a result of the simulation study, it is suggested that the unbiased and precise estimators of $g(0)$ and total density of whale group were obtained from the $g(0)$ estimation model of the current study.

The estimators of $g(0)$ without covariates were widely scattered from the true values, against the estimators from covariates were precise. Doi et al. (1983) indicated that effort of observation differs by the angle between track line and the target. Barlow (2001) presented that perpendicular distance was affected by group size, Beaufort scale, cue, vessel, year and survey area. Murase et al. (2004) reported that detection distance was shorter when Beaufort scale was larger and cue was body than brow. Although our simulation expressed somewhat extreme situations, these reports and our result suggested that the covariates should be considered as far as no-IO estimation model.

A discrete model was used for the estimation. One of the advantages of discrete model was the flexibility of the model. Even if the problem was complicated, the discrete model could describe it flexibly. Although it takes more time to calculate using the discrete model than analytical models, it is not serious problem by using recent powerful PC.

For further study, the validation or sensitivity of the assumption of Q(0,0) and Δs should be examined. Moreover, the estimation of parameters concerning with the covariates is thought to be more robust if it is estimated from the all year pooled data in stead of year by year. Furthermore, the other covariates such as visibility etc should be incorporated for further study.

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Table 1. Effective data number obtained from experiment of chasing behavior research in JARPA. The data was used to calculate the surfacing interval p.d.f, $f_s(\tau, s)$. 2.5% and 97.5% represent the lower or upper of 95% confidence interval of average.

Size	*N*	Average	2.5%	97.5%
1	223	131.0	114.3	147.8
2	177	114.0	96.7	131.3
3	123	55.3	41.9	68.6
5	134	46.4	39.4	53.4
Total	657			

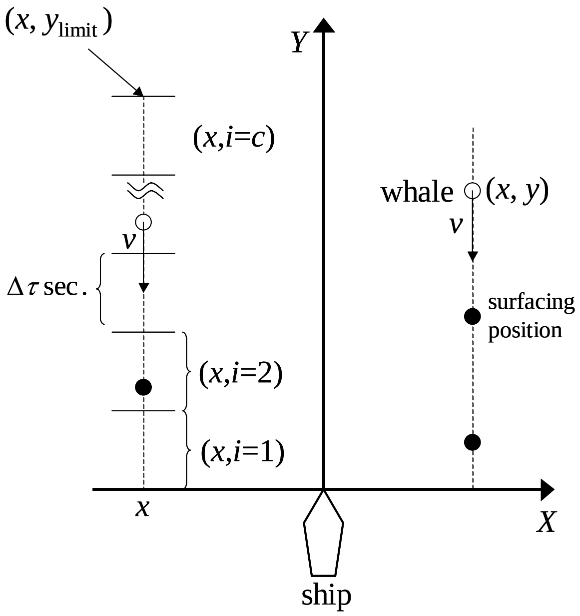

Fig. 1. **Assumption of model. X and Y axis are relative coordinate that represents a poision of whale group from a research ship. A whale at (*x*, *y*) moves to the inverse direction of Y axis with ship velocity *v*. An open circle represents diving whale position and close circle surfacing. y_{limit} means the limit distance of detection. In this study, a segment line coordinate, (x, *i*) *i*={1,2,…,*c*}, was defined, where *c* is number of segments between y=0 to y=y_{limit}. The whale on border of a segment line can pass the segment for sec. calculated by *v* and unit of segment line length.**

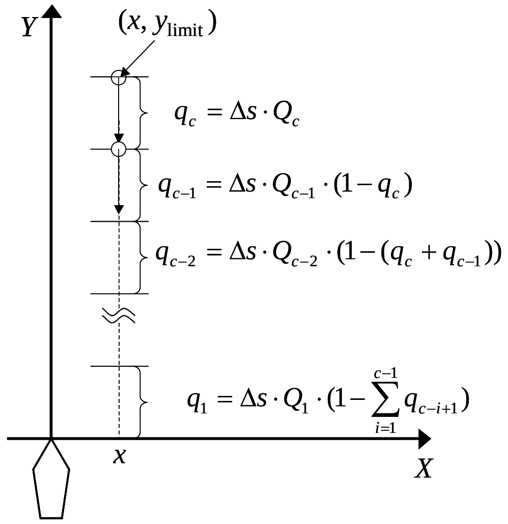

Fig. 2. **Calculation process of detection probability q_i. A whale group farter than (x, y_{limit}) can't detect and the surfacing probability between a segment line is constant Δs. Therefore the detection probability q_c for first segment $i=c$ is calculated by product of Δs and Q_c that is detection probability when whale is surfacing. Furthermore, the probability $q_{c\text{-}1}$ for the next segment, $i=c$-1, is calculated by product of Δs, Q_c and undetected whale rate, 1-q_c.**

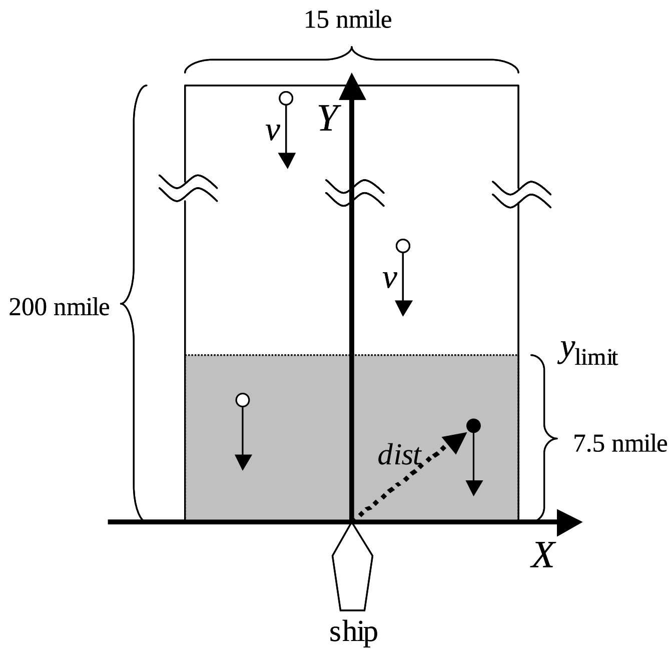

Fig. 3. **The define field of research area in the data generator of simulation. Whales distribute randomly in strip area and move to the inverse direction of Y axis with ship velocity v. Ship is fixed at Origin, O. A Research ship can detect the surfacing whales into the gray zone with detection probability $Q(dist \mid \mathbf{H})$**

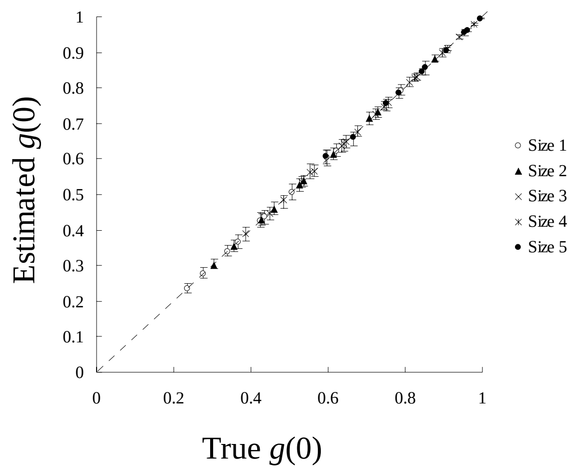

Fig. 4. **Result of estimated $g(0)$s from true Q model under each 50 covariates combination. Error bar means 25-75%^{ile}.**

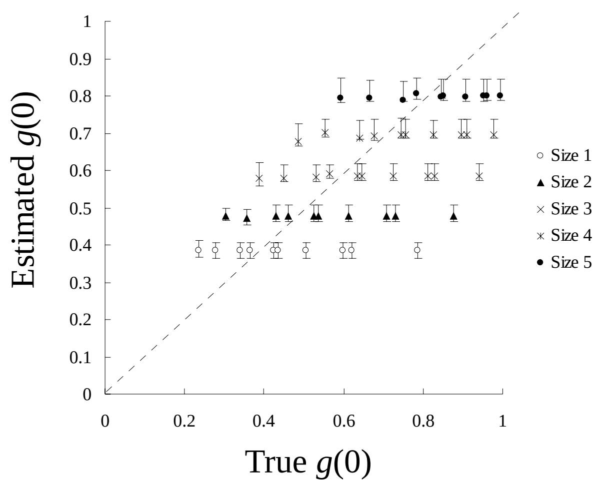

Fig.5. **Result of estimated $g(0)$s from limited Q model under 50 covariates combination. Error bar means 25-75%^{ile}.**

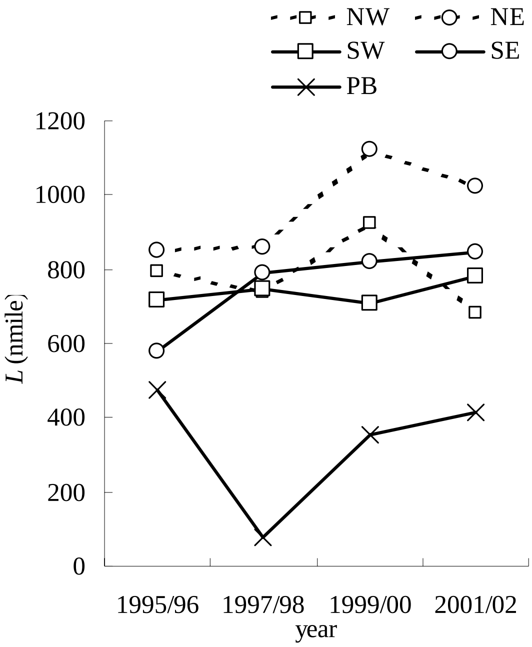

Fig. 6. Survey line length of each year and stratum.

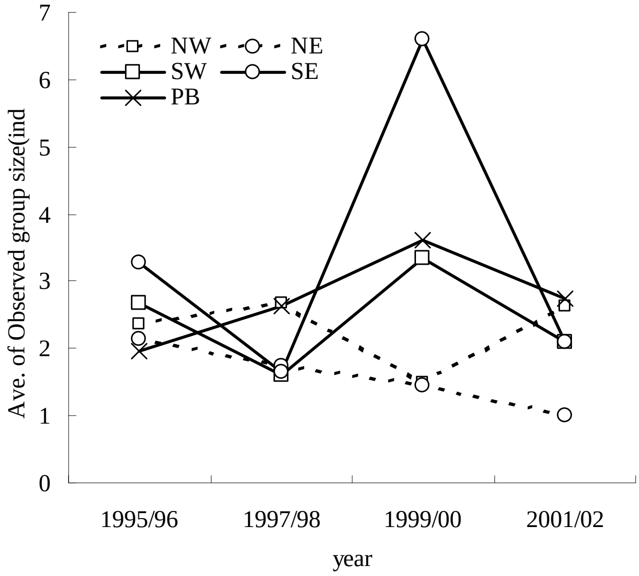

Fig. 7. Average of detected group size.

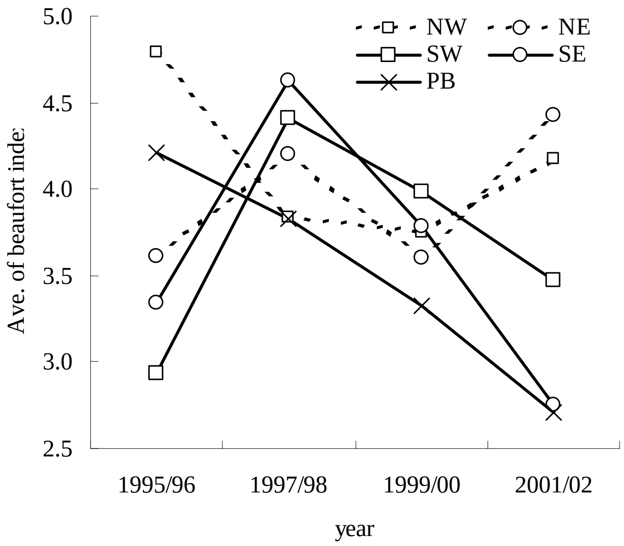

Fig. 8. Average of Beaufort index.

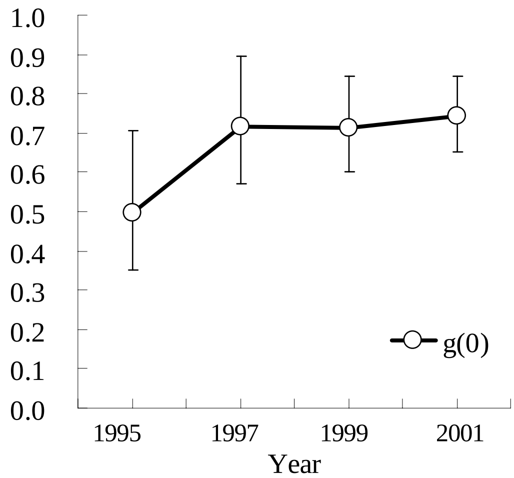

Fig. 9. Estimated g(0) from the data of JARPA in area IV by using the proposed method in the current study.

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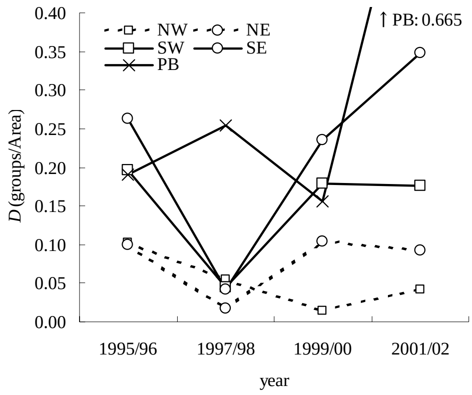

Fig. 10. Estimated density of groups.

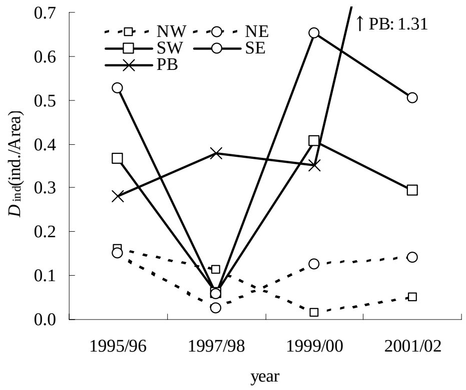

Fig. 11. Estimated density of individuals.

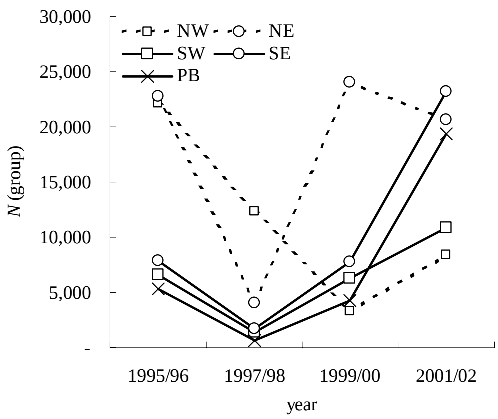

Fig. 12. Number of group.

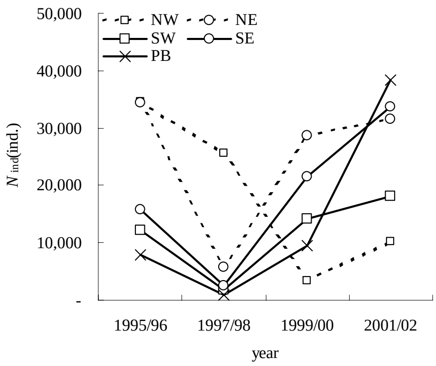

Fig. 13. Number of individual.

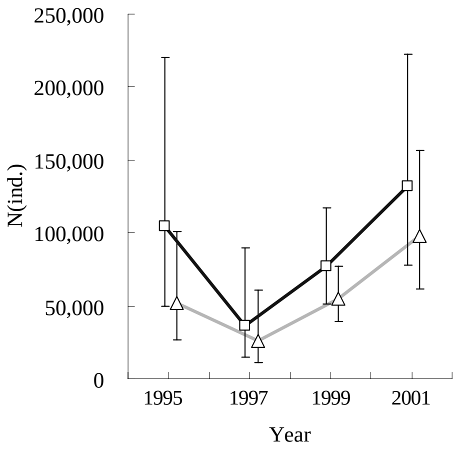

Fig. 14. Estimated population of Antarctic minke whale in area IV from JARPA data. Black line indicate the estimated population of the current study, and the gray line is that of the traditional line transect method (Hakamada et al. 2001). Error bars shows 95% confidence interval.

APPENDIX

Δs was calculated only from surfacing interval time of whale. It was assumed that an observer can detect one group by a cue (blow or surfaced body) that was made by a member of the group.

The surfacing interval time was defined as the time interval of cues made by a group. Probability density function of surfacing interval time is defined as $f_{\mathrm{s}}(\tau, s \mid \mathbf{A})$, where τ is the surfacing interval time, s is the group size and $\mathbf{A}$ is the parameter vector. The cumulative probability density function is denote as $F_{\mathrm{s}}(\tau, s \mid \mathbf{A})$. The ratio of groups that are diving for time τ is given as $1-F_{\mathrm{s}}(\tau, s)$. The probability that the group surface once in the interval of $\Delta\tau$ subject to the group is diving at time τ was given as follows;

$$\frac{F_{\mathrm{s}}(\tau+\Delta\tau, s)-F_{\mathrm{s}}(\tau, s)}{1-F_{\mathrm{s}}(\tau, s)} \tag{17}$$

The ratio that a group has stayed diving for time t at any time is proportional to $1-F_{\mathrm{s}}(t, s)$. Then probability density of diving time t is described as follows;

$$\frac{1-F_{\mathrm{s}}(t, s)}{\int_0^{\infty}\{1-F_{\mathrm{s}}(\tau, s)\}d\tau} \tag{18}$$

From these equations and assumptions, Δs is calculated by formula of expectation below;

$$\begin{aligned}
\Delta s(s) &= \int_0^{\infty}\frac{1-F_{\mathrm{s}}(\tau, s)}{\int_0^{\infty}\{1-F_{\mathrm{s}}(\tau, s)\}d\tau}\cdot\frac{F_{\mathrm{s}}(\tau+\Delta\tau, s)-F_{\mathrm{s}}(\tau, s)}{1-F_{\mathrm{s}}(\tau, s)}d\tau \\
&= \frac{\int_0^{\infty}\{F_{\mathrm{s}}(\tau+\Delta\tau, s)-F_{\mathrm{s}}(\tau, s)\}d\tau}{\int_0^{\infty}\{1-F_{\mathrm{s}}(\tau, s)\}d\tau} \\
&= \frac{\int_0^{\Delta\tau}\{1-F_{\mathrm{s}}(\tau, s)\}d\tau}{\int_0^{\infty}\{1-F_{\mathrm{s}}(\tau, s)\}d\tau}
\end{aligned} \tag{19}$$

Furthermore expectation of $\tau \geq 0$ equals

$$E[\tau \mid s] = \int_0^{\infty}\{1-F_{\mathrm{s}}(\tau, s)\}d\tau \tag{20}$$

It follows that Δs is described

$$\Delta s(s) = \frac{\int_0^{\Delta\tau}\{1-F_{s}(\tau, s)\}d\tau}{E[\tau \mid s]} \tag{21}$$