PROGRESS ON APPLICATION OF ADAPT-VPA TO MINKE WHALES IN AREAS IV AND V GIVEN UPDATED INFORMATION FROM IDCR/SOWER AND JARPA SURVEYS

M. Mori and D. S. Butterworth
MARAM (Marine Resource Assessment and Management Group)
Department of Mathematics and Applied Mathematics
University of Cape Town, Rondebosch 7701, South Africa

ABSTRACT

The ADAPT-VPA assessment methodology of Butterworth *et al*. (1999) is applied to abundance estimates (from both IDCR/SOWER and JARPA surveys) and catch at age data (both commercial and scientific) for Areas IV and V. The methodology is extended to be able to take account of inter-annual differences in the distribution of the population between the two Areas when they are assessed jointly. An important feature of these updated results is that revised JARPA estimates of abundance are shown to be statistically comparable with estimates from the IDCR/SOWER programme (i.e. calibration factor not significantly different from 1). The general pattern shown by results is of a minke whale abundance trend that increased over the middle decades of the 20th Century to peak at about 1970, and then declined for the next three decades. The recruitment trend is similar, though with its peak slightly earlier. The factor to which the results are most sensitive is the value of natural mortality *M*. The assessments do show retrospective patterns, primarily related to changes in the best estimate of *M* as time has progressed. This in turn seems linked to the IDCR/SOWER survey trends suggesting higher, and the JARPA survey trends lower estimates of *M*. For the assessment of the two Areas combined, *M* is estimated at 0.068 with a CV of 0.12; this compares with CVs of typically 0.35 for the Area-specific assessments of Butterworth *et al*. (1999), which were based on eight seasons' fewer data. The paper reflects an account of work in progress, and suggestions are made of areas where further analyses would be desirable.

INTRODUCTION

Butterworth *et al*. (1999) applied an ADAPT-like version of Virtual Population Analysis (VPA) to commercial and research (JARPA) catches-at-age, together with estimates of abundance from IDCR/SOWER and JARPA surveys, to estimate past trends in recruitment and abundance of the minke whale populations in Antarctic Areas IV and V. The data available for those analyses extended to the 1995 season (note that for convenience in this paper, Antarctic seasons will usually be referenced by the earlier of the two years concerned, so that the 1995/96 season is termed 1995 here). Those analyses focused on Area IV, and used a 3-year-3-age grouping of the data to facilitate estimation. Butterworth *et al*. (2002) extended those analyses using data available to the 1999 season.

This paper reports progress on extending those analyses yet further to take account of the most recent (including to the 2003 season) minke whale data and analyses now available from the JARPA programme, specifically:

- Updated catch-at-age data from the research samples, kindly provided by Zenitani.

- Updated estimates of abundance from the JARPA surveys, kindly provided by Hakamada.

Although the analysis method used for this paper is essentially the same as that of Butterworth *et al*. (1999), there are some important changes (primarily as regards application):

- The computations are now implemented using the ADMB package, which provides more powerful and reliable minimization capabilities when fitting the ADAPT-VPA model to the data and estimating parameters.
- Given this greater power, together with the extended data set, it has become feasible to focus on a conventional 1-year-1-age analysis, instead of the previous grouping of data into 3-year-3-age blocks.
- Although analyses are conducted for Areas IV and V separately, the primary focus of the paper is upon a joint analysis of both Areas; this treats the two as containing a single stock that distributes itself in these two feeding regions in a manner which admits some variability from year to year.

It is to be stressed that this paper reports “work in progress”. Hence in some respects the results shown are not considered to be finalized “most appropriate model formulations”, though the authors consider that subsequent refinements are unlikely to lead to quantitatively substantial changes to these results.

DATA

Tables 1a and 1b list the catch-at-age matrices used for Areas IV and V respectively for these analyses. These reflect commercial catches from 1971 to 1986, and scientific research catches from 1987 to 2003. The commercial catch information is unchanged from that used in Butterworth *et al*. (1999). The scientific catch information has been developed as described in Butterworth *et al*. (1999), but the underlying ageing information incorporates some subsequent revisions by Zenitani.

Tables 2a and 2b list the survey estimates of abundance for Areas IV and V respectively that are used in the analyses, together with the associated survey sampling CVs. Estimates from the IDCR/SOWER surveys were taken from Branch (2003), except for 1998 which was developed by T. Branch (pers. commn) as detailed below. These estimates exclude consideration of like-minke whale sightings and the areas considered are comparable between the estimates, being extended northward to 60°S as shown in Branch (2003). The following process was used to obtain the abundance estimate for Area IV from the third circumpolar set of cruises that is shown in Table 2a:

1) The 1998/99 estimate from Branch (2003) was used for the stratum from 80° to 130°E.
2) The 1994/95 estimates from Branch and Butterworth (2001) were added to the estimate from 1) for the following strata: all of Prydz stratum, one-half of the EN stratum, and one-half of the ES stratum. The IO and closing estimates then need to be inverse-variance weighted, and the value of R=0.826 (CV=0.089) for pseudo-passing estimates, obtained in Branch and Butterworth (2001) was used for this.

The estimates from the JARPA surveys listed in Table 2 were provided by Hakamada. It should be noted that a revised method of analysis was used to obtain these; this incorporates estimated calibration factors to render these estimates comparable to surveying by vessels responsible for sightings only ("SV" vessels, as distinct from "SSV" vessels which also sample the whales). This in turn should render these JARPA-based abundance estimates more comparable to those from the IDCR/SOWER surveys.

METHODOLOGY

Readers are referred to Butterworth *et al*. (1999) for finer details of the ADAPT-VPA methodology applied. This section focuses on the main features of the approach only, and in particular specifies the changes involved in moving from a 3-year-3-age to a 1-year-1-age basis for analysis. The new feature of the analyses which admits inter-annual variation in the distribution of whales between Areas IV and V when these two Areas are assessed in combination is also described.

Population model

The basic population dynamics are taken to be governed by the equations:

$$N_{y+1,a+1} = \left(N_{y,a} - C_{y,a}\right) \cdot e^{-M_a} \qquad 1 \le a \le m \tag{1}$$

$$F_{y,a} = C_{y,a} \big/ N_{y,a} \tag{2}$$

where

$N_{y,a}$ is the number of minke whales (here of both sexes combined) of age *a* present at the start of year *y*;

$C_{y,a}$ is the number of such whales taken during year *y* (note that the ADAPT methodology, as applied here, assumes the data provided for these catches-at-age in Table 1 to be known without error);

M_a is the (possibly age-dependent) rate of natural mortality;

$F_{y,a}$ is the proportion of the animals of age *a* present at the start of year *y* that are taken (the "fishing proportion"); and

m is the oldest age considered in the model-fitting process.

Consistent with Butterworth *et al*. (1999), most of the analyses of this paper take m=30. An argument for this approach is that samples sizes for large ages are very small, and furthermore older ages are less reliably determined because of the closer spacing of earplug rings. However, results are also shown for alternative choices for m up to 45. For analysis purposes, natural mortality M_a is presumed infinite at age 45 and above, so that animals captured above this age are ignored. For choices of m < 45, results are projected forward from age m to age 45 using equation (1) and known catches, so that all the analyses take account of minke whales up to age 45 irrespective of the choice made for m.

A key aspect of the parameterization of the ADAPT-VPA model applied is the assumption that the fishing proportion F is separable (in expectation). Different selectivity patterns are assumed for the years of commercial and scientific catches:

$$F_{y,a}^{E} = \begin{cases} S_a^c F_y^E & y \le 1986 \\ S_a^s F_y^E & y \ge 1987 \end{cases} \tag{3}$$

where

S_a^c is the selectivity-at-age for the period of commercial catches (S_m^c =1);

S_a^s is the selectivity-at-age for the period of scientific catches (S_m^s =1);

F_y^E is the fishing proportion (in expectation) for year y on age m (i.e. the fully selected fishing proportion in cases where $S_a^{c/s} \le 1$); and

$F_{y,a}^E$ is the expected fishing proportion on animals of age a for year y; this differs from the actual proportion $F_{y,a}$ because actual catches $C_{y,a}$ differ from their expectations ($C_{y,a}^E = F_{y,a}^E N_{y,a}$) because of sampling variability.

The primary estimable parameters of the model are effectively:

- The natural mortality M_a (usually taken to be age-independent).
- The oldest-age numbers-at-age $N_{y,m}$.
- The most-recent-year numbers-at-age $N_{n,a}$, where n is the last year for which data are available.

Given values for these parameters, the complete numbers-at-age matrix ($N_{y,a}$) for the population can then be computed by use of equation (1).

The Likelihood function

For single Area assessments, the likelihood function has three components related to the IDCR/SOWER estimates of abundance, the JARPA estimates of abundance and the catch-at-age data. The contribution of the first of these to the negative of the log likelihood (ignoring constants) is given by:

$$-\ln L_1 = \sum_y \frac{1}{2\sigma_y^2}\left(\ln N_y^{obs} - \ln \hat{N}_y\right)^2 \tag{4}$$

where

N_y^{obs} is the abundance estimate for year y;

σ_y is the standard error of the logarithm of N_y^{obs}, which is approximated by $\sqrt{CV_y{}^2 + CV_{add}{}^2}$;

CV_y is the survey sampling CV estimated for N_y^{obs};

CV_{add} is an additional CV to reflect the fact that survey sampling error is not the only factor contributing to the difference between N_y^{obs} and $\hat{N}_y$; and

$\hat{N}_y$ is the model estimate of 1+ abundance for year *y*, given by:

$$\hat{N}_y = \sum_{a=1}^{45} \hat{N}_{y,a} \tag{5}$$

The contribution of the JARPA estimates of abundance is similar, except that these are treated as indices of relative abundance:

$$-\ln L_2 = \sum_y \frac{1}{2\sigma_y^2} \left(\ln N_y^{obs} - \ln\left(q\hat{N}_y\right)\right)^2 \tag{6}$$

where

q is the multiplicative bias associated with abundance estimates from JARPA compared to those from IDCR/SOWER, and is given by its maximum likelihood estimate:

$$\ln \hat{q} = \left\{ \sum_y \frac{\ln\left(N_y^{obs} / \hat{N}_y\right)}{\sigma_y^2} \right\} \Bigg/ \left\{ \sum_y 1/\sigma_y^2 \right\} \tag{7}$$

Finally the contributions of the commercial and the scientific catch-at-age data are given by:

$$-\ln L_3^c = -\lambda^c \sum_{y=1971}^{1986} \sum_{a=16}^{m} C_{y,a}^* \ln \hat{\rho}_{y,a} \tag{8}$$

$$-\ln L_3^s = -\lambda^s \sum_{y=1987}^{2003} \sum_{a=1}^{m} C_{y,a}^* \ln \hat{\rho}_{y,a} \tag{9}$$

where

$C_{y,a}^*$ is the effective number of animals of age *a* caught during year *y*, computed as $C_{y,a} C_y^* / C_y$;

C_y is the total catch in numbers during year *y*;

C_y^* is the number of animals actually aged for year *y*, which also are taken into account in the L_3 calculation for that year (i.e. with ages from 16 to *m* for the commercial, and from 1 to *m* for the scientific catches);

$\lambda^{c/s}$ is a factor to account for overdispersion in the commercial/scientific catch-at-age distribution (underdispersion is not admitted, so that $0 < \lambda \le 1$); and

$\hat{\rho}_{y,a}$ is the model-estimate of the expected proportion of the catch in year *y* that consists of animals of age *a*, which from equation (3) is given by:

$$\hat{\rho}_{y,a} = \begin{cases} S_a^c N_{y,a} \Big/ \sum_{a'=16}^{m} S_{a'}^c N_{y,a'} & y \le 1986 \\ S_a^s N_{y,a} \Big/ \sum_{a'=1}^{m} S_{a'}^s N_{y,a'} & y \ge 1987 \end{cases} \tag{10}$$

A consistent selectivity-at-age pattern (S_a^c) is assumed to apply only above age 15 on the basis of arguments by Sakuramoto and Tanaka (1985) that below this age the pattern varies appreciably from year to year. The overdispersion factors λ are estimated by iterative application of the formula:

$$\lambda^{c/s} = \sum_y 1 \Bigg/ \sum_y \left\{ \frac{C_y^* \sum_a \left(\rho_{y,a} - \hat{\rho}_{y,a}\right)^2}{\sum_a \hat{\rho}_{y,a}\left(1 - \hat{\rho}_{y,a}\right)} \right\} \tag{11}$$

where the years and ages in the summations are as adopted above for L_3^c and L_3^s, and $\rho_{y,a}$ is the observed proportion of the catch during year *y* which consists of animals of age *a*:

$$\rho_{y,a} = \begin{cases} C_{y,a}^* \Big/ \sum_{a'=16}^{m} C_{y,a'}^* & y \le 1986 \\ C_{y,a}^* \Big/ \sum_{a'=1}^{m} C_{y,a'}^* & y \ge 1987 \end{cases} \tag{12}$$

When Areas IV and V are assessed in combination, allowance needs to be made for the fact that the survey estimates now apply to only a portion of the combined IV+V minke whale abundance. If the proportion in Area IV in year *y* is p_y, and hence the proportion in Area V that year is $\left(1 - p_y\right)$, then equation (4) is adjusted to read:

$$-\ln L_1 = \sum_{y(IV)} \frac{1}{2\sigma_y^2}\left[\ln N_y^{obs} - \ln\left(p_y \hat{N}_y\right)\right]^2 + \sum_{y(V)} \frac{1}{2\sigma_y^2}\left[\ln N_y^{obs} - \ln\left\{\left(1 - p_y\right)\hat{N}_y\right\}\right]^2 \tag{13}$$

where the two summations are over years with IDCR/SOWER surveys in Area IV and in Area V respectively. Equation (5) for the contribution from the JARPA survey abundance estimates is adjusted similarly. The p_ys become estimable parameters of the model, though note that in years with a survey in both Areas, the same p_y is taken to apply (as any difference arising from JARPA and IDCR/SOWER surveys taking place at slightly different times during the season seems likely to be relatively small).

Allowing the p_y to be unconstrained (other than that $0 \le p_y \le 1$) would lead to an under-determined model, in the sense that the p_ys could then adjust for the model to match each abundance estimate exactly (except in years with surveys in both Areas). On the other hand, setting $p_y = p$ (constant) seems unrealistic as it does not allow for changes in the distribution of whales between the two Areas from year to year. Accordingly the p_y s have been assumed to follow a beta distribution:

$$p_y \sim \mathrm{B}(\alpha, \beta) \tag{14}$$

with the approximate estimation approach then used (within the MLE context applied) being the addition of the following further contribution to the negative of the log likelihood:

$$-\ln L_4 = \sum_y \left[\ln \Gamma(\alpha) + \ln \Gamma(\beta) - \ln \Gamma(\alpha+\beta) - (\alpha-1)\ln p_y - (\beta-1)\ln(1-p_y)\right] \tag{15}$$

where the summation extends over the years for which there is a survey in at least one of the two Areas.

In implementation, the parameter:

$$p = \frac{\alpha}{\alpha+\beta} \tag{16}$$

which is the average proportion of the combined abundance to be found in Area IV is treated as an estimable parameter of the model. The parameter α is fixed externally, with different values being chosen to reflect different levels of inter-annual variability (in terms of a CV) of p:

$$CV(p) = \sqrt{\frac{\beta}{[\alpha(\alpha+\beta+1)]}} \tag{17}$$

RESULTS

Base Case specification: Areas IV+V combined

The selectivity forms assumed for the Base Case assessment follow those adopted for the analyses of Butterworth *et al*. (1999):

$$S_a^c = \begin{cases} S^c & a = 16,...,21 \\ 1 & a = 22,...,m \end{cases} \tag{18}$$

$$S_a^s = \begin{cases} S^s & a = 1,...,6 \\ 1 & a = 7,...,m \end{cases} \tag{19}$$

where S^c and S^s (both constrained to be ≤ 1) are estimated in the model fitting process. Arguments for the appropriateness of setting selectivity to be flat at older ages for both the commercial and the scientific catches are advanced in Butterworth *et al*. (1999).

The remaining specifications for the Base Case assessment are:

i) m=30

ii) $M_a = M$ (constant)

iii) q=1

iv) CV(p)=0.2 (and CV_{add}=0).

The first two of these specifications maintain the approach of earlier papers (Butterworth *et al*. 1999, 2002) for consistency. The justification for the others is most easily provided by reference to the results for the IV+V combined assessments provided in Table 3a. This Table includes results for the case where the bias q of the JARPA surveys relative to the IDCR/SOWER surveys is estimated rather than fixed as input. The estimate is close to 1 ($\hat{q} = 1.025$) and certainly not significantly different from it, so the choice was made to treat the two sets of surveys as comparable (i.e. to fix $q = 1$) for the Base Case.

The basis for selecting CV(p)=0.2 is a little more complex. If the proportion of the population p_y in Area IV is constant over time, one finds that the differences between the observed abundances and those estimated by the ADAPT-VPA model are typically larger than the survey sampling CVs for the survey would suggest (see the case "p constant" in Table 3a – this difference is measured by the standard deviation of the standardized residuals – SDSR — which at 1.82 substantially exceeds 1 in this case). We account for this discrepancy by assuming that it is a consequence of the variation in the proportions of the population that migrate to Area IV and to Area V each year, and increase the extent of this variability (i.e. increase CV(p)) until the SDSR for abundance estimates drops to 1. This is achieved for the choice CV(p)=0.2, which also reflects consistency with the variability of the specific p_y values estimated (note SDSR(p) for the Base Case is 0.93 – very close to 1). An alternative way of accounting for the discrepancy would be to entertain a positive value of CV_{add} (see equation 4). As CV_{add} and CV(p) effects are confounded, CV_{add} has been set to zero here.

Base Case results: Areas IV+V combined

Fig. 1 shows the estimated commercial and scientific selectivities at age, while Fig. 2 shows the associated residuals to the fit to the catch-at-age proportions. These do not appear to reflect any obvious systematic patterns, except that virtually all predicted proportions of 1-year-olds in the scientific catches are too low, indicating a need to estimate S_1^s separately to allow it to be less than, rather than force it to be equal to S_2^s.

Fig. 3a shows the fits of the ADAPT-VPA model to the 1+ abundance estimates for Areas IV and V separately for each Area, together with the estimates for the proportions (p_y and $1-p_y$) of the total population in each Area each year. The abundance plots include the survey estimates of abundance together with their survey sampling related estimated 95% confidence intervals, nearly all of which are consistent with the overall trends estimated.

A feature of 1+ abundance plots of Fig. 3a which needs further explanation is that both "estimated" and "adjusted" trajectories are shown. The total population estimates sum numbers-at-age ($N_{y,a}$) present each year over ages 1 to 45 (see equation 5). However, limitation of the analyses to m=30 and to data from years 1971 to 2003 means that the earliest cohort for which recruitment ($N_{y,1}$) is estimable is the 1942 cohort ($N_{1942,1}$), and further that numbers at age a=45 are available only from year 1986 onwards. The "estimated" total abundance, which excludes contributions to N_y from cohorts not included in the assessment which are treated as zero, are hence negatively biased. To "adjust" them to attempt to remove this bias, a log-linear regression of recruitment *vs* year over the period 1945 to 1968 is used to estimate recruitment levels before 1942 back to 1930, at which time an unexploited equilibrium age-structure is assumed to apply. This then allows values to be generated for the "missing" $N_{y,a}$ s, for a better representation of the total population size in the earlier years.

Fig. 3b shows estimated recruitment ($N_{y,1}$) and total population size trajectories for Area IV+V combined. The 95% confidence intervals about these trajectories are also shown (though these do not include allowance for the "adjustment" of the total population size for earlier years). These intervals are displayed both for the case where natural mortality *M* is estimated, and where it is fixed at its maximum likelihood estimate. The purpose is to show that precision is fairly good for all years if *M* is assumed known; however, the estimation of *M*, though having little impact on precision for recent decades, results in increasing imprecision as one goes further back in time.

The Base Case assessment manifests values of λ^c =0.89, λ^s =0.53, i.e. slight overdispersion.

Results of Sensitivity tests for Areas IV+V combined

The sensitivity tests listed in Table 3a are generally self-explanatory. A number of associated Figures, which primarily compare the estimated trends in recruitment and total population size between these tests, are also shown:

- Fig. 4 – different choices for CV(*p*) – variability in interannual distribution between Areas IV and V.
- Fig. 5 – different choices for the maximum age *m*.
- Fig. 6 – different choices for age-independent natural mortality *M*.
- Fig. 7 – retrospective assessment for 2, 5 and 8 years earlier.
- Fig. 8 – consequences of omitting either the JARPA or the IDCR/SOWER estimates of abundance.
- Fig. 9 – different choices for selectivity slopes.
- Fig. 10 – confidence intervals for M_a when a linear trend over *a*=1 to 30 is admitted.

Other tests in Table 3a consider a different distributional form for p_y, estimating *q* and a 33% negative bias in abundance estimates (a consequence, perhaps, of the survey analysis assumption that *g*(0)=1). Some factors are tested in combination with increasing *m* from 30 to 45.

The selectivity slope tests are of two types. First only commercial selectivity is assumed to have a linear trend (fixed or estimated) above age *a*=21 (i.e. instead of being flat with $S_a^c = 1$ as in equation 18) ; S_{30}^c is kept fixed at 1 for these results, while S_a^s remains at 1 for *a*≥7. For the "Animals in the pack ice" test, the assumption is made that selectivity changes in the same way with age for both commercial and scientific catches because, for example, of older animals being preferentially found in that area, and so not being available for capture (for the case $S_{26}^c > 1$). In these cases the trend in S_a^s continues down to age *a*=10.

Specification of Area-specific assessments

To maintain comparability with the Areas IV+V combined assessment, the Base Case specifications for both Areas treated on their own set m=30, $M_a = M$ (constant) and $q = 1$. The justification for the last choice is somewhat problematic for Area IV, as a variant of the Base Case which estimates q provides an estimate significantly larger than 1 (see Table 3b). However, the 1998 IDCR/SOWER abundance estimate, which is unusually low, is very influential in such an analysis. If it is omitted from consideration, q again becomes not significantly different from 1, as is the case for Area V.

The (original) Base Cases maintain CV_{add}=0 as for the Areas IV+V combined assessment. For both Areas IV and V, this leads to the SDSR for the abundance estimates fitted exceeding 1. CV_{add} has been increased to reduce the value of SDSR to 1. Only a rather small CV_{add} is needed to achieve this for Area V, so that the associated Base Case has not been amended. However, a rather larger effect is evident for Area IV, so that the plots of results shown in Fig. 11 are for the Revised Base Case (CV_{add}=0.2) in the case of Area IV.

Results of Base Case and Sensitivity tests for Area-Specific assessments

Fig. 11 compares these Base Case results with those for the Areas combined Base Case assessment. For the last three decades, the Area specific assessments show a total population size which is decreasing more steeply for Area IV, but flatter for Area V. Corresponding M estimates are lower for Area V than for Area IV.

Other sensitivity tests conducted for the Area-specific assessment include a 3-year-3-age grouping analysis for comparison with the results of Butterworth *et al*. (1999), and retrospective analyses.

DISCUSSION

The consistent pattern shown by virtually all the assessments considered, whether for Area IV and V in combination or separately, is of minke whale abundance that increased in the middle decades of the 20 th Century to peak at about 1970, and then declined for the next three decades. Recruitment trends show a similar pattern, peaking a few years earlier than total abundance.

Of the various sensitivity tests to the Areas combined assessment, four show relatively minor effects in terms of trend estimates:

- Negative bias in estimates of abundance in absolute terms.
- A larger CV on the proportion in Area IV (p_y) distribution gives a steeper historic increase in recruitment and a flatter peak in total abundance (Fig. 4).
- Increasing m, the largest age considered in the likelihood, leads to steeper historic increases in recruitment and abundance (Fig. 5).
- If selectivity slopes at larger ages are treated as estimable rather than set flat, the estimates themselves are not greatly different from those when flatness is assumed (Fig. 9).

The parameter to which trends are the most sensitive is natural mortality *M*. A larger *M* means a lesser historic increase in abundance, and a steeper current decline (Fig. 6). Retrospective analyses indicate a decrease in the estimate of *M* as time has progressed (Fig. 7). This links to the relatively greater influences of the JARPA estimates of abundance as their number has increased with time: the IDCR/SOWER estimates tend to favor a higher *M*, and the JARPA a lower *M*, with consequential impacts on historic recruitment and population trends (Fig. 8).

Comparing with earlier assessments using this methodology (Butterworth *et al*. 1999, 2002), CVs on estimates of *M* for Area-specific assessments as data have accumulated have decreased roughly speaking from 0.35 to 0.3 to 0.2. For the current Areas combined assessment, *M* is about 0.07 with a CV of about 0.12. The considerable increase in precision for these analyses compared to the earlier ones is closely linked to the revised JARPA estimates of abundance. These now seem comparable to those from the IDCR/SOWER programme, and this assists in improving precision (the CV for *M* increases from 0.12 to 0.17 if *q* has to be estimated, see Table 3a).

There is statistically significant evidence (compare $-\ln L$ values in Table 3a) for some increase in *M* as age increases (Fig. 10).

Comparing the current Area-specific assessments with earlier analyses, the indications are of estimated historic increase rates in recruitment that have remained low for Area IV, whereas for Area V originally high values have decreased somewhat. Retrospective analyses for the current assessment reflect a trend of estimates of *M* that have decreased as time has progressed (as in the case of the Areas combined assessments).

FURTHER WORK

The parameterization of selectivity factors adopted for these analyses has followed that of Butterworth *et al*. (1999). The data available then could justify only relatively parsimonious formulations. With more data now available, this aspect needs to be revisited and more flexible functional forms explored (e.g. logistic forms which allow also for exponential trends at higher ages).

Detection of significant evidence for a trend in natural mortality *M* with age merits further attention, including consideration of more realistic forms then the simple linear trend considered here.

The estimates of overdispersion factors for the Base Case Areas combined assessment (λ^c =0.89, λ^s =0.53) are a little surprising, particular the greater over-dispersion indicated for the scientific catch-at-age data given that it derives from an attempt at random sampling. This merits further examination, and is possibly linked to the absence of any account being taken of ageing error in this analysis.

The estimation procedure used for the p_y parameters is somewhat *ad hoc.* However, integrating over these random effects would be computationally onerous. A more straightforward approach might be to extend the whole analysis to a fully Bayesian form, integrating over priors for all the parameters.

IN SUMMARY

The combined analysis of catch at age and survey abundance estimates provides considerable and important insight into the dynamics of minke whales in Areas IV and V over the middle and final decades of the 20th century. Information from the JARPA programme has played a key role in improving the precision of estimates of these trends as time has progressed (and with it the precision with which natural mortality *M* can be estimated).

ACKNOWLEDEMENTS

The authors thank R. Zenitani, T. Hakamada and T. Branch for the provision of data and analysis results used in this paper, and A. E. Punt for advice on the ADMB code utilized.

REFERENCES

Branch, T. A. 2003. Updated circumpolar abundance estimates for Southern Hemisphere minke whales including results from the 1998/99 to 2000/01 IDCR-SOWER surveys. Report of the Scientific Committee, Annex G, Appendix 3. *Journal of Cetacean Research and Management* (Suppl.) 5:271-275.

Branch, T. A., and D. S. Butterworth. 2001. Southern Hemisphere minke whales: standardised abundance estimates from the 1978/79 to 1997/98 IDCR/SOWER surveys. *Journal of Cetacean Research and Management* 3:143-174.

Butterworth, D. S., Punt, A. E., Geromont, H. F., Kato, H. and Fujise, Y. 1999. Inferences on the dynamics of Southern Hemisphere minke whales from ADAPT analyses of catch-at-age information. *Journal of Cetacean Research and Management* 1: 11-32.

Butterworth, D. S., Punt, A. E., Branch, T. A., Fujise, Y., Zenitani, R. and Kato, H. 2002. Updated ADAPT VPA recruitment and abundance trend estimates for Southern Hemisphere minke whales in Areas IV and V. IWC Document SC/54/IA25 (20pp).

Sakuramato, K. and Tanaka, S. 1985. A new multi-cohort method for estimating Southern Hemisphere minke whale populations. *Rep. int. Whal. Commn* 35: 261-271.

Table 1a Catch at age matrix for Area IV (see text for source details). For economy of space, ages have been grouped by 3, so that age 5 (for example) combines ages 4-6. Note that 1971 reflects the 1971/72 season.

Year/Age	2	5	8	11	14	17	20	23	26	29	32	35	38	41	44	47	50	54+
1971	121	256	314	310	352	319	284	235	146	95	99	45	30	25	3	7	17	4
1972	286	842	890	665	580	457	307	262	104	82	40	26	7	9	0	0	0	0
1973	495	535	620	691	529	427	398	229	213	182	93	65	62	6	12	13	0	0
1974	39	131	263	360	314	377	189	232	154	82	37	19	23	3	2	0	7	0
1975	14	110	127	176	139	140	71	33	23	23	10	6	9	0	0	0	0	0
1976	21	163	180	329	323	225	135	134	32	21	21	0	4	6	0	0	6	0
1977	46	56	131	128	156	164	77	66	69	34	17	6	11	0	0	2	0	0
1978	36	111	177	237	229	182	143	103	63	34	29	15	2	2	2	0	4	0
1979	85	174	180	222	241	268	189	139	104	79	58	39	27	19	14	15	6	0
1980	92	208	269	271	305	300	264	228	127	114	50	63	35	25	20	8	7	0
1981	65	155	195	221	211	239	171	119	106	59	41	15	17	5	4	3	1	0
1982	65	120	160	253	229	347	288	199	132	81	28	23	14	20	4	4	2	0
1983	105	225	252	334	269	300	205	188	108	76	15	9	2	7	0	0	0	0
1984	32	82	124	190	212	193	196	172	128	67	45	30	0	9	8	0	0	0
1985	14	52	97	136	192	218	211	201	107	82	33	28	31	2	0	2	0	0
1986	21	47	109	152	193	265	192	166	122	85	38	33	20	10	7	3	0	0
1987	28	44	33	24	29	25	31	14	17	13	3	6	2	1	0	0	0	0
1988	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1989	35	53	48	23	15	36	30	19	27	14	8	7	5	5	1	0	0	0
1990	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1991	39	39	39	24	20	21	18	12	16	20	15	9	7	2	2	2	1	1
1992	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1993	58	47	46	42	22	25	16	11	17	12	14	7	6	1	1	2	1	1
1994	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1995	34	35	34	50	26	17	20	18	26	12	12	17	7	5	3	7	3	4
1996	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1997	87	36	36	27	20	19	14	17	10	10	15	13	10	2	5	6	0	2
1998	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1999	40	42	37	32	28	28	22	17	10	15	17	11	13	8	7	3	0	0
2000	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2001	42	39	38	21	26	24	27	17	14	18	9	16	14	10	8	3	1	2
2002	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2003	39	30	45	21	28	30	23	16	12	13	18	8	18	7	8	8	4	1

Table 1b Catch at age matrix for Area V (see text for source details). For economy of space, ages have been grouped by 3, so that age 5 (for example) combines ages 4-6. Note that 1974 reflects the 1974/75 season.

Year/Age	2	5	8	11	14	17	20	23	26	29	32	35	38	41	44	47	50	54+
1974	21	62	129	133	131	89	74	48	30	3	10	4	0	0	0	0	0	0
1975	12	83	75	110	99	95	76	38	26	7	4	4	0	0	0	0	0	0
1976	10	130	150	204	286	225	145	94	76	67	27	26	7	9	9	0	0	2
1977	15	66	149	175	117	119	99	56	44	23	8	4	2	5	3	0	0	0
1978	9	19	47	72	69	72	17	43	32	19	4	9	0	4	0	0	0	0
1979	40	74	153	143	174	231	139	86	118	74	49	22	7	6	14	15	0	0
1980	10	86	123	129	152	162	108	116	109	105	32	50	30	13	13	9	0	0
1981	14	86	99	112	168	188	152	103	106	51	32	31	14	8	7	4	2	0
1982	26	113	216	258	282	295	244	175	117	72	48	16	17	12	3	1	0	1
1983	78	182	164	214	251	195	130	98	62	27	24	13	3	3	1	0	0	0
1984	24	66	88	104	112	173	128	84	56	39	13	14	5	1	1	0	0	0
1985	33	52	66	102	128	182	128	105	101	44	31	24	6	7	1	2	1	0
1986	4	32	48	102	117	141	171	125	118	80	31	25	5	6	1	2	1	3
1987	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1988	13	34	27	28	39	25	21	15	8	10	5	5	5	0	0	0	0	0
1989	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1990	4	39	30	24	35	39	42	30	23	13	12	10	9	3	4	4	0	0
1991	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1992	22	20	39	41	27	19	21	29	32	23	23	15	11	2	2	2	0	0
1993	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1994	32	23	44	35	39	31	28	20	21	17	11	16	8	4	0	0	0	0
1995	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1996	21	24	17	40	38	21	28	21	26	18	21	13	12	15	6	2	6	0
1997	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1998	39	13	38	20	40	30	20	20	30	22	14	19	9	5	2	4	1	1
1999	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2000	16	26	29	31	33	42	9	26	28	21	18	16	15	2	6	3	4	5
2001	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2002	22	33	28	37	33	37	29	22	22	16	19	8	8	3	5	2	3	1
2003	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Table 2 Abundance estimates from sightings surveys (see text for source details).

a) Area IV

Survey	Year to which applied		Estimate (CV)
	3year-3age	1year-1age	
IDCR 1978/79	1980	1978	134308 (0.173)
IDCR 1988/89	1989	1988	60180 (0.184)
IDCR 1998/99 + part 1994/95	1998	1998	18619 (0.154)
JARPA 1989/90	1989	1989	54539 (0.215)
JARPA 1991/92	1992	1991	54959 (0.243)
JARPA 1993/94	1995	1993	41943 (0.215)
JARPA 1995/96	1995	1995	42134 (0.220)
JARPA 1997/98	1998	1997	32656 (0.252)
JARPA 1999/00	2001	1999	49867 (0.169)
JARPA 2001/02	2001	2001	68503 (0.167)
JARPA 2003/04	2004	2003	47858 (0.358)

b) Area V

Survey	Year to which applied		Estimate (CV)
	3year-3age	1year-1age	
IDCR 1980/81	1980	1980	257767 (0.280)
IDCR 1985/86	1986	1985	290724 (0.140)
IDCR 1991/92	1992	1991	190081 (0.180)
JARPA 1990/91	1992	1990	195743 (0.210)
JARPA 1992/93	1992	1992	122048 (0.229)
JARPA 1994/95	1995	1994	168566 (0.268)
JARPA 1996/97	1998	1996	171332 (0.261)
JARPA 1998/99	1998	1998	198423 (0.233)
JARPA 2000/01	2001	2000	179796 (0.210)
JARPA 2002/03	2004	2002	226884 (0.161)

Table 3a Results for the Base Case and sensitivity tests for the assessment of Areas IV+V combined. $N_{y,1}$ reflects recruitment and N_y total abundance. Increase rates are given as annual proportions. The contributions to the total negative log-likelihood (-lnL(total)), in terms of the notation in the text, are –lnL(CAA)=-lnL_3, -lnL(indices)=-lnL_1-lnL_2, -ln$L(p_y)$=-lnL_4. Chi2= χ^2 is defined in equation 20 of Butterworth *et al*. (1999). The value of $\bar{p} = \alpha/(\alpha + \beta)$ is the estimated average proportion of the whales to be found in Area IV. SDSR is the standard deviation of the standardized residuals; for the abundance indices the standardization is in terms of the survey sampling CVs for each survey, whereas for the proportion p_y in Area IV it is in terms of the standard deviation assumed to apply to this distribution. Estimated CVs, where given, are based upon the Hessian approximation – this was cross-checked against likelihood profile estimation for the CV of *M*, and found to reflect good accuracy.

Area IV+V																	
Scenario	Ny,1 incr.rate 45-68	$N_{83,1}/N_{68,1}$	$N_{lstyr-5,1}/N_{68,1}$	M (CV)	Ny incr.rate 80-lstyr	-lnL (total)	-lnL (CAA)	-lnL (indices)	-lnL (Py)	Chi2/DF	$\bar{p}$	SDSR (abun)	SDSR (p)	Others			
"Base Case"	0.046	0.569	0.204	0.068 (0.123)	-0.027	314.156	341.423	10.590	-37.856	1.006	0.185	1.004	0.929				
Log Normal (s=0.2)	0.046	0.561	0.197	0.069 (0.125)	-0.028	346.026	341.401	10.301	-5.676	1.009	0.201	1.075	0.883				
p constant	0.038	0.503	0.161	0.077 (0.094)	-0.035	376.405	341.690	34.715	-	0.978	0.198	1.818	-				
Beta (CV=0.1)	0.041	0.530	0.178	0.074 (0.104)	-0.032	310.662	341.560	21.542	-52.440	1.006	0.182	1.432	0.827				
Beta (CV=0.4)	0.052	0.618	0.238	0.062 (0.156)	-0.021	319.387	341.310	5.012	-26.935	1.008	0.184	0.691	0.755	*q*			
q estimated	0.045	0.559	0.197	0.070 (0.167)	-0.028	314.145	341.430	10.738	-38.023	1.006	0.185	1.011	0.915	1.025 (0.149)			
m=35	0.059	0.554	0.201	0.068 (0.122)	-0.026	389.808	417.072	10.497	-37.761	1.006	0.185	1.000	0.934				
m=45	0.066	0.543	0.201	0.068 (0.122)	-0.026	482.712	510.057	10.412	-37.758	0.963	0.185	0.996	0.934				
Abundance x 1.5	0.045	0.577	0.203	0.070 (0.118)	-0.028	314.524	341.572	10.786	-37.834	1.007	0.185	1.014	0.930				
Mfix=0.05	0.063	0.709	0.308	0.050	-0.011	317.100	341.512	12.017	-36.428	1.005	0.184	1.070	1.019				
Mfix=0.09	0.026	0.437	0.126	0.090	-0.045	316.591	341.812	13.203	-38.424	1.007	0.186	1.121	0.869	Mmax (CV)	Mmin (CV)		
M linear	0.035	0.570	0.222	0.064	-0.027	311.578	339.226	10.376	-38.024	1.008	0.185	0.994	0.920	0.097 (0.155)	0.031 (0.583)		
M linear (*m*=45)	0.056	0.536	0.204	0.062	-0.026	478.512	505.860	10.519	-37.867	0.965	0.185	1.001	0.928	0.104 (0.142)	0.040 (0.319)		
Retrospective (lstyr=2001)	0.050	0.492	0.323	0.079 (0.121)	-0.037	289.370	314.678	7.532	-32.840	1.008	0.197	0.890	0.924				
Retrospective (lstyr=1998)	0.015	0.345	0.130	0.101 (0.116)	-0.058	245.807	272.083	5.000	-31.277	1.015	0.182	0.791	0.573				
Retrospective (lstyr=1995)	0.015	0.339	0.174	0.101 (0.164)	-0.058	273.413	238.366	13.249	21.798	1.054	0.324	1.486	0.841				
Omit JARPA data	-0.070	0.258	0.038	0.124 (0.132)	-0.083	329.478	341.463	1.755	-13.741	0.988	0.182	0.765	0.417				
Omit IDCR data	0.082	0.921	0.571	0.03 (0.005)	0.010	310.605	340.904	3.431	-33.729	1.001	0.190	0.676	0.414				
														Sc16-21	Sc22	Sc26	Sc45
Sc26=0.80 (Sc30=1)	0.084	0.561	0.202	0.068 (0.125)	-0.023	327.907	354.815	10.564	-37.472	1.021	0.186	1.003	0.946	0.532	0.600	0.800	
Est Select Slope (Sc30=1)	0.039	0.577	0.208	0.068 (0.125)	-0.027	313.687	340.995	10.577	-37.886	1.006	0.185	1.004	0.928	1.056	1.121	1.060	
Sc26=1.20 (Sc30=1)	0.024	0.597	0.221	0.066 (0.128)	-0.027	315.444	342.835	10.514	-37.904	1.010	0.185	1.001	0.926	1.355	1.400	1.200	
Sc26=1.40 (Sc30=1)	0.010	0.628	0.242	0.063 (0.135)	-0.027	321.581	349.074	10.382	-37.875	1.029	0.185	0.994	0.926	1.802	1.800	1.400	
Sc26=0.95 (*m*=45)	0.075	0.542	0.205	0.067 (0.125)	-0.024	488.161	515.400	10.371	-37.609	0.979	0.186	0.994	0.941	0.758	0.900	0.950	1.188
Sc26=1.05 (*m*=45)	0.058	0.548	0.200	0.069 (0.120)	-0.027	478.053	505.450	10.469	-37.866	0.949	0.185	0.999	0.929	0.939	1.103	1.050	0.807
Sc26=1.10 (*m*=45)	0.051	0.554	0.201	0.069 (0.119)	-0.027	474.806	502.219	10.514	-37.928	1.001	0.185	1.001	0.926	1.023	1.190	1.100	0.643
Est Select Slope (*m*=45)	0.047	0.558	0.202	0.069 (0.119)	-0.027	473.098	500.517	10.534	-37.953	0.937	0.185	1.002	0.925	1.071	1.248	1.124	0.535
Sc26=1.15 (*m*=45)	0.055	0.526	0.191	0.070 (0.118)	-0.027	487.262	514.731	10.488	-37.957	0.976	0.185	0.999	0.925	0.906	1.291	1.150	0.453
Sc26=0.80 (*m*=45, Sc30=1)	0.090	0.537	0.208	0.065 (0.128)	-0.022	500.104	527.129	10.352	-37.377	1.026	0.186	0.993	0.950	0.543	0.600	0.800	
Est Select Slope (*m*=45, Sc30=1)	0.065	0.544	0.201	0.068 (0.122)	-0.026	482.643	510.000	10.425	-37.782	0.962	0.185	0.996	0.933	0.873	1.036	1.018	
Sc26=1.20 (*m*=45, Sc30=1)	0.049	0.554	0.200	0.069 (0.118)	-0.028	487.717	515.115	10.560	-37.959	0.963	0.184	1.003	0.925	1.159	1.400	1.200	
Animals in the pack ice (Sc30=1)																	
Sc26=Ss26=0.9	0.040	0.650	0.256	0.093 (0.094)	-0.024	319.477	347.216	10.081	-37.820	1.019	0.185	0.980	0.932	0.715	0.800	0.900	
Est Select Slope	0.045	0.535	0.184	0.060 (0.176)	-0.028	313.539	340.600	10.730	-37.844	1.010	0.185	1.013	0.930	1.062	1.121	1.060	
Sc26=Ss26=1.1	0.044	0.517	0.175	0.056 (0.147)	-0.028	313.759	340.705	10.883	-37.829	1.008	0.185	1.018	0.930	1.150	1.200	1.100	

Table 3b Results for the Base Case and sensitivity tests for the assessment of Area IV and of Area V each treated as closed populations. Notation is as for Table 3a.

Area IV											
Scenario	Ny,1 incr.rate 45-68	N83,1/N68,1	N98,1/N68,1	M (CV)	Ny incr.rate 80-lstyr	-lnL (total)	-lnL (CAA)	-lnL (indices)	Chi2/DF	SDSR (abun)	*q*
Original "Base Case"	0.020	0.422	0.129	0.090 (0.115)	-0.042	238.654	213.759	24.895	0.934	2.128	
q estimate	0.040	0.320	0.079	0.106 (0.106)	-0.059	231.392	213.748	17.644	0.934	1.791	1.740 (0.070)
omit IDCR 1998	0.033	0.510	0.201	0.076 (0.140)	-0.027	222.057	213.360	8.697	0.932	1.319	
omit IDCR 1998, q estimate	0.047	0.643	0.314	0.062 (0.245)	-0.012	221.024	213.213	7.811	0.932	1.250	0.716 (0.326)
m =45	0.030	0.424	0.131	0.081 (0.128)	-0.037	378.141	352.444	25.698	0.961	2.162	
3yr-3age	0.019	0.280	0.105*	0.095 (0.112)	-0.048	57.389	31.796	25.592	0.946	2.157	
Retrospective (lstyr=2001)	0.015	0.403	0.273	0.095 (0.115)	-0.044	223.863	200.287	23.576	0.937	2.172	
Retrospective (lstyr=1998)	-0.029	0.213	0.040	0.141 (0.091)	-0.085	185.987	182.205	3.782	0.969	0.972	
Retrospective (lstyr=1995)	-0.003	0.290	0.230	0.115 (0.145)	-0.062	169.052	168.776	0.276	0.974	0.303	
Cvadd =0.2 (Revised Base Case)	0.029	0.468	0.161	0.080 (0.195)	-0.033	223.628	213.031	10.597	0.931	0.988	
Area V											
Scenario	Ny,1 incr.rate 48-68	N83,1/N68,1	N98,1/N68,1	M (CV)	Ny incr.rate 80-lstyr	-lnL (total)	-lnL (CAA)	-lnL (indices)	Chi2/DF	SDSR (abun)	*q*
"Base Case"	0.067	0.628	0.267	0.051 (0.202)	-0.017	233.134	227.094	6.040	1.147	1.099	
q estimate	0.085	0.856	0.495	0.030 (0.464)	0.004	231.682	227.096	4.586	1.147	0.958	0.697 (0.255)
m =45	0.077	0.613	0.289	0.054 (0.191)	-0.015	341.545	335.865	5.680	1.135	1.066	
3yr-3age	0.060	0.604	0.330	0.050 (0.268)	-0.016	46.266	39.664	6.602	1.250	1.149	
Retrospective (lstyr=2001)	0.054	0.508	0.171	0.065 (0.196)	-0.031	213.738	209.923	3.815	1.144	0.921	
Retrospective (lstyr=1998)	0.026	0.360	0.237	0.087 (0.209)	-0.052	194.901	192.777	2.124	1.155	0.779	
Retrospective (lstyr=1995)	0.021	0.318	0.098	0.092 (0.227)	-0.060	166.291	164.624	1.667	1.200	0.770	
Cvadd =0.02	0.062	0.629	0.268	0.051 (0.204)	-0.017	233.078	227.104	5.973	1.147	0.995	

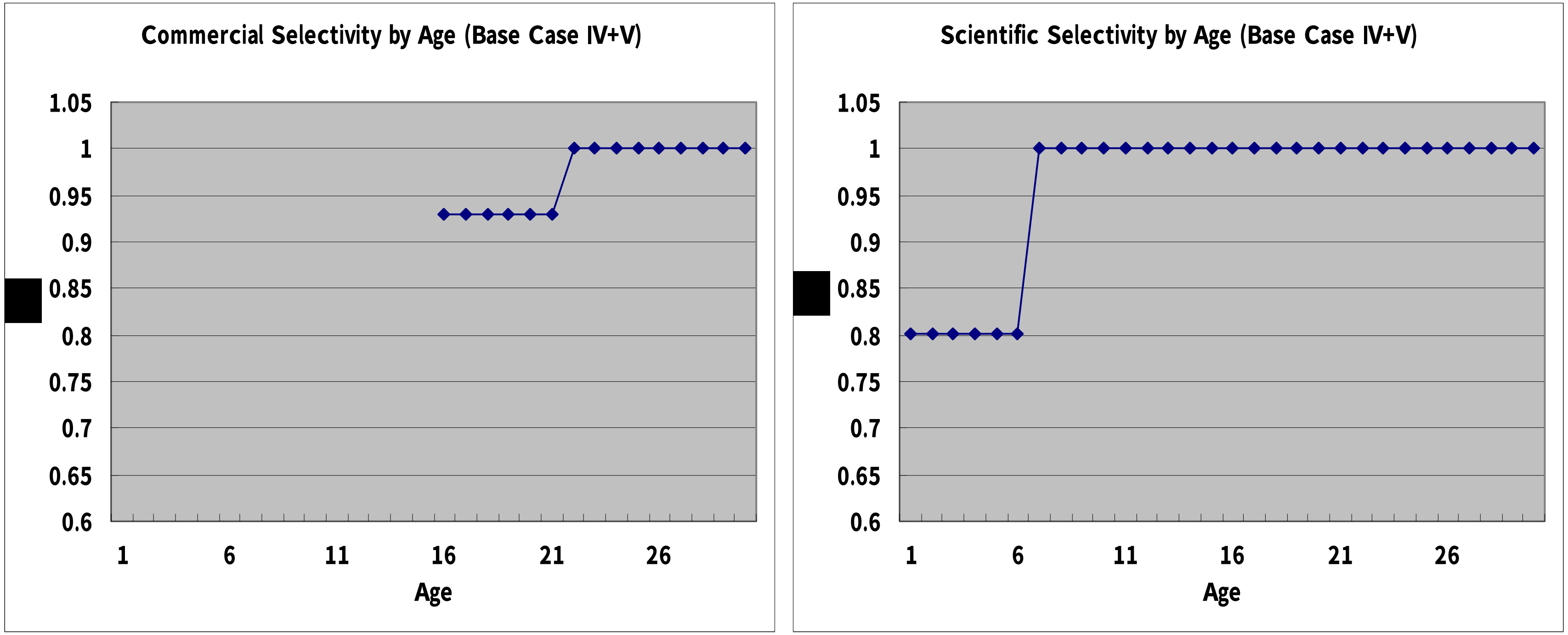

Figure 1. Estimated commercial and scientific selectivities by age for the "Base Case" for Areas IV+V combined.

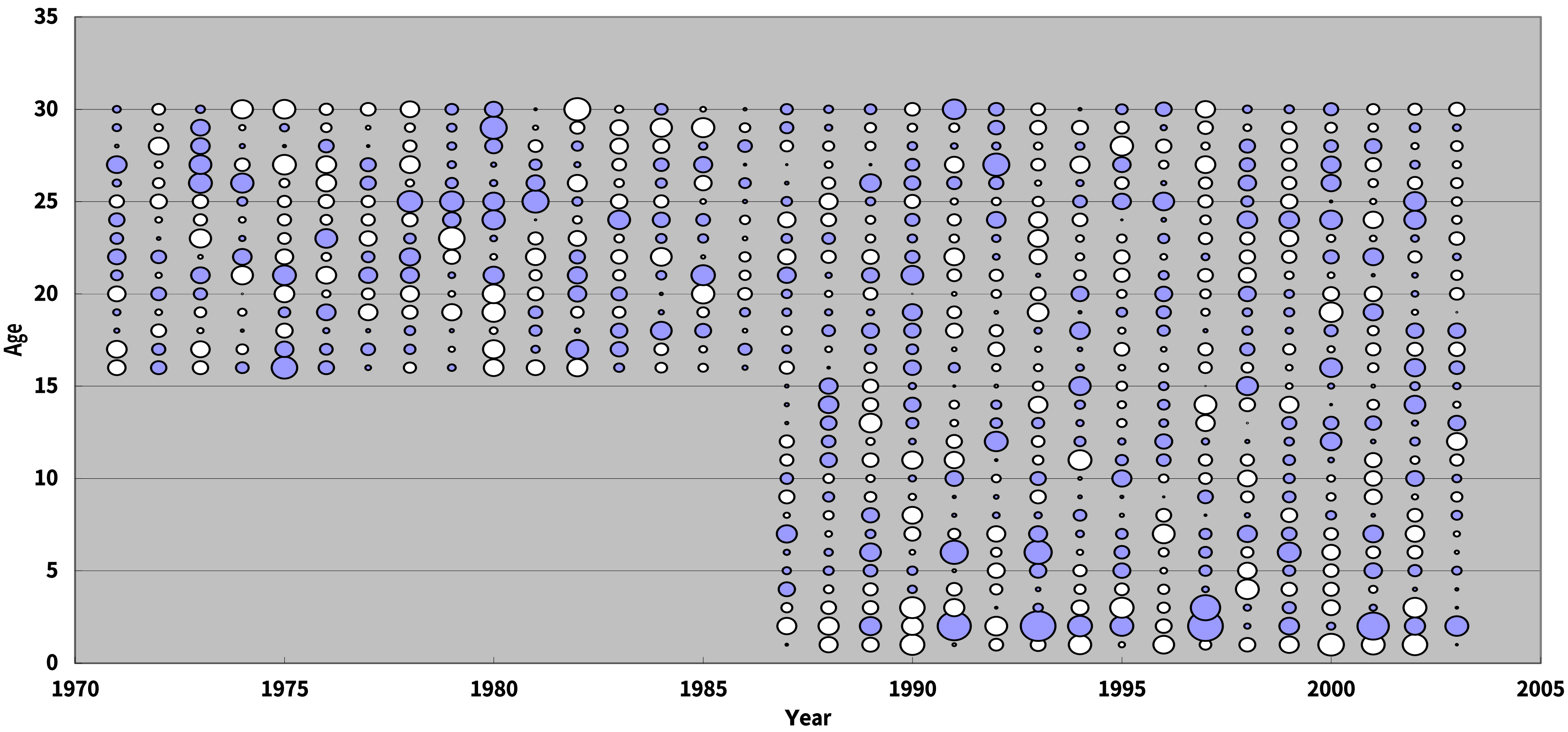

Figure 2. Standardised residuals of the observed and predicted catch at age proportions for the Area IV+V combined Base Case, shown as a bubble plot. Gray bubbles are positive and white bubbles negative residuals. The size of the bubble indicates the magnitude of the residual. Standardised residuals are defined by : $\sqrt{C^*_{y,a}\lambda_y}\,\frac{\rho_{y,a}-\hat{\rho}_{y,a}}{\sqrt{\hat{\rho}_{y,a}}}$.

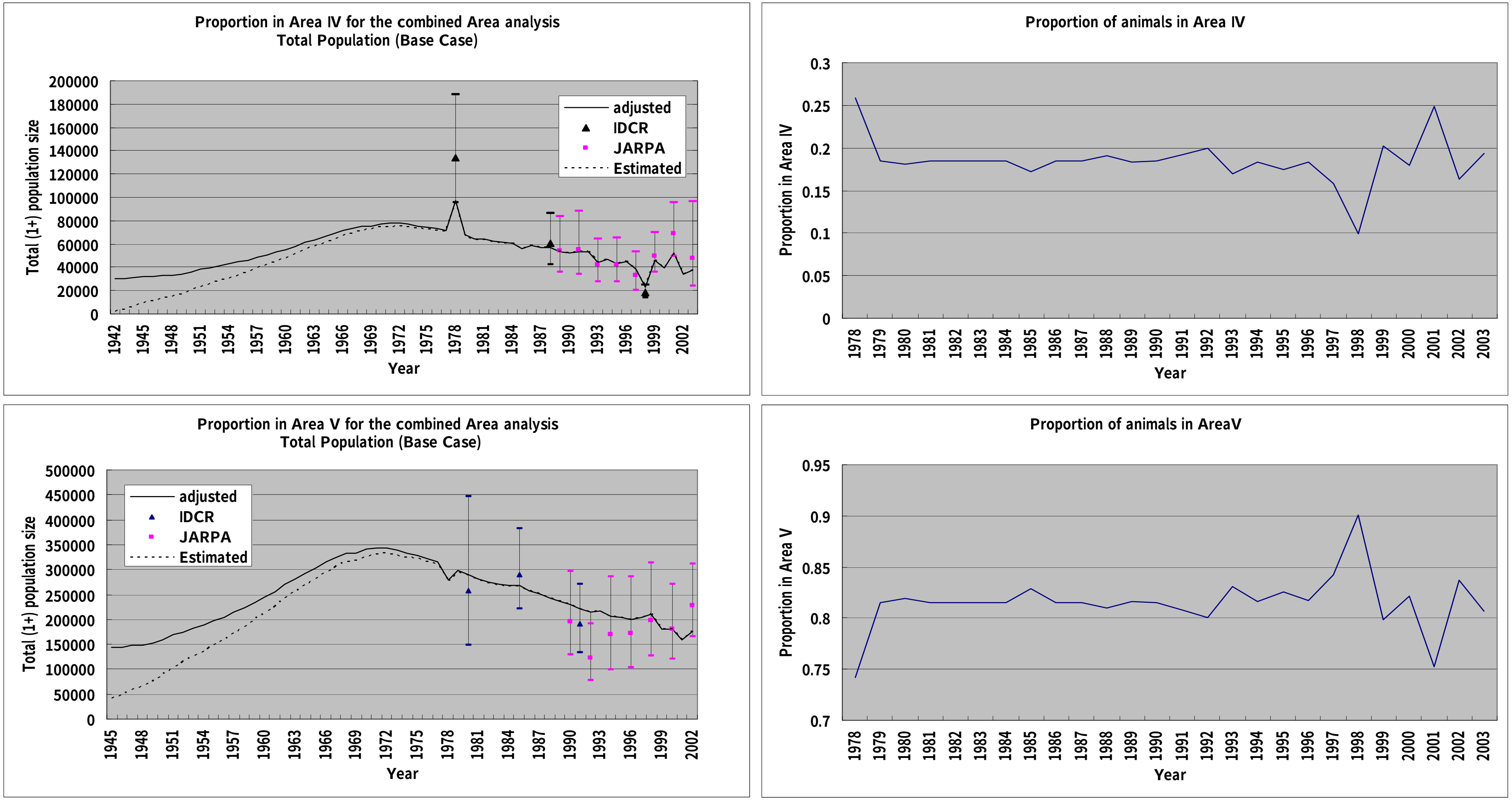

Figure 3a. Fits of the ADAPT-VPA model to the total (1+) abundance estimates for Areas IV and V separately for each Area, together with the estimates for the proportions (p_y and $1-p_y$) of the total population in each Area each year for the Area IV+V combined Base Case. The abundance plots include the survey estimates of abundance together with their survey sampling estimated 95% confidence intervals. The difference between the "estimated" and "adjusted" total population sizes is explained in the text.

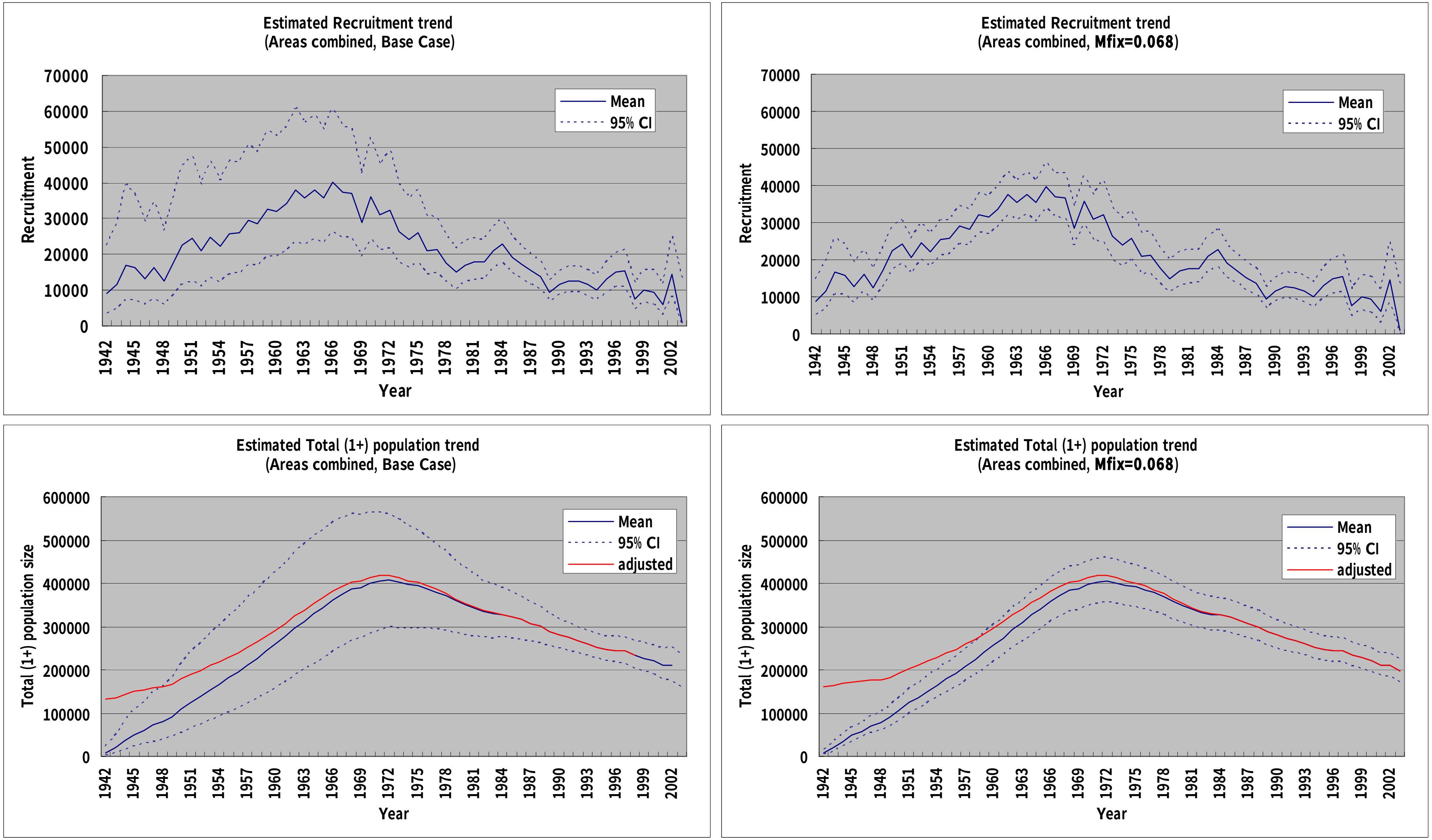

Figure 3b. Estimated recruitment ($N_{y,1}$) and total population size trajectories for Areas IV+V together for the Areas IV+V combined Base Case. In the righthand plots, natural mortality M is fixed at its maximum likelihood estimate.

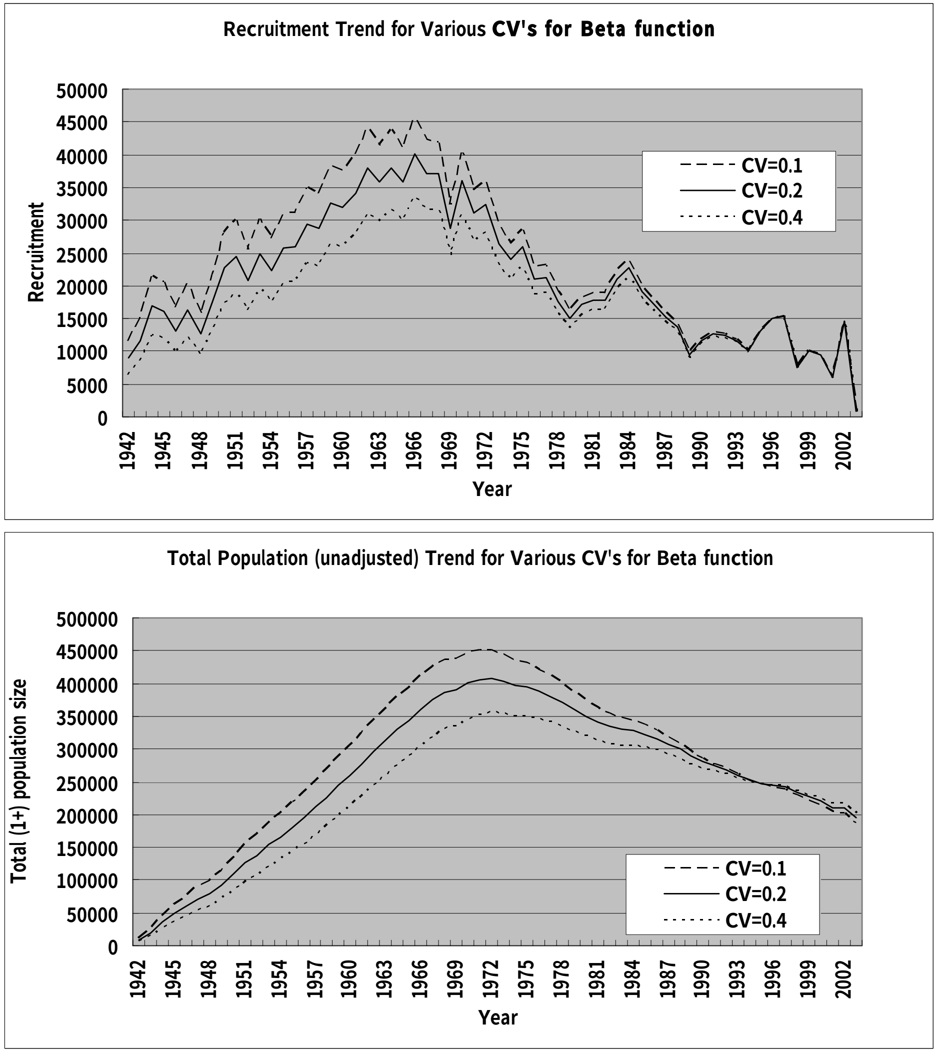

Figure 4. Estimated recruitment ($N_{y,1}$) and total population size trajectories for different choices of CVs for the p_y distribution for the Areas IV+V combined assessment.

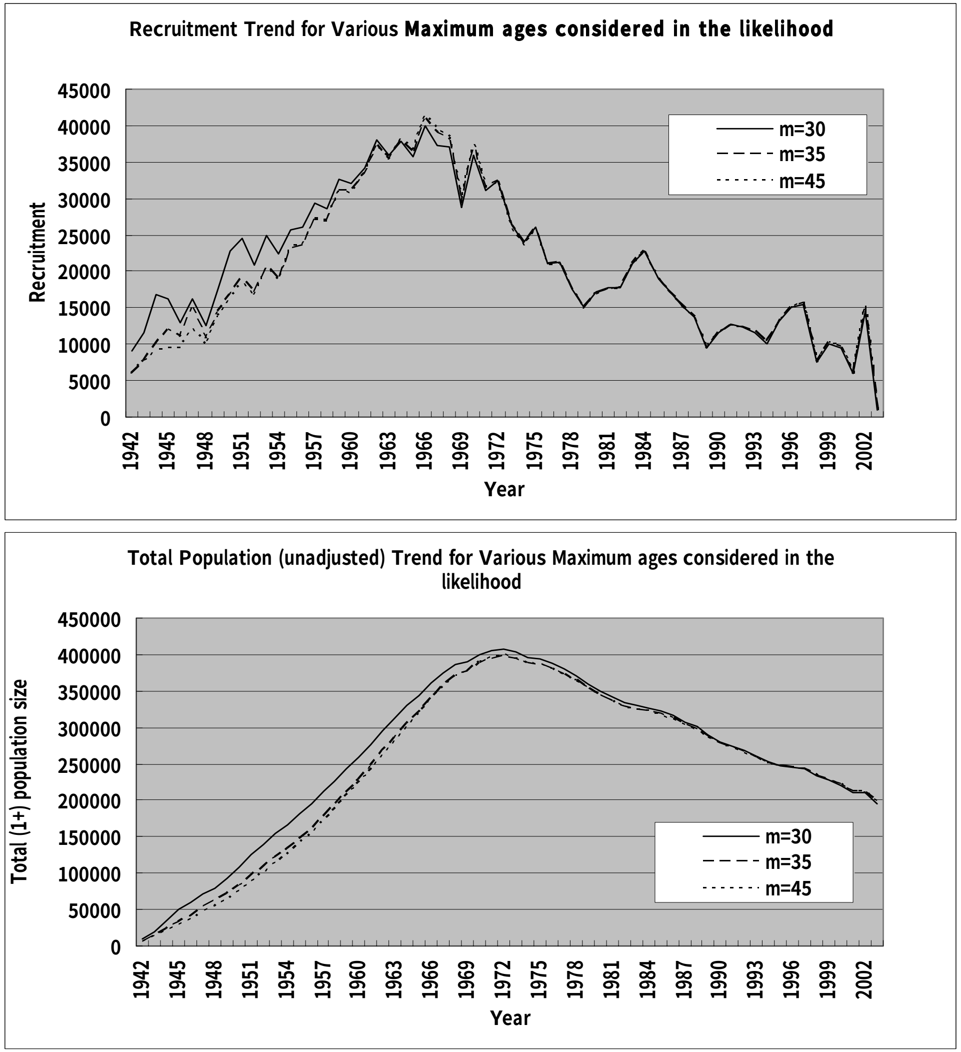

Figure 5. Estimated recruitment ($N_{y,1}$) and total population size trajectories for different choices of maximum age *m* for the Areas IV+V combined assessment.

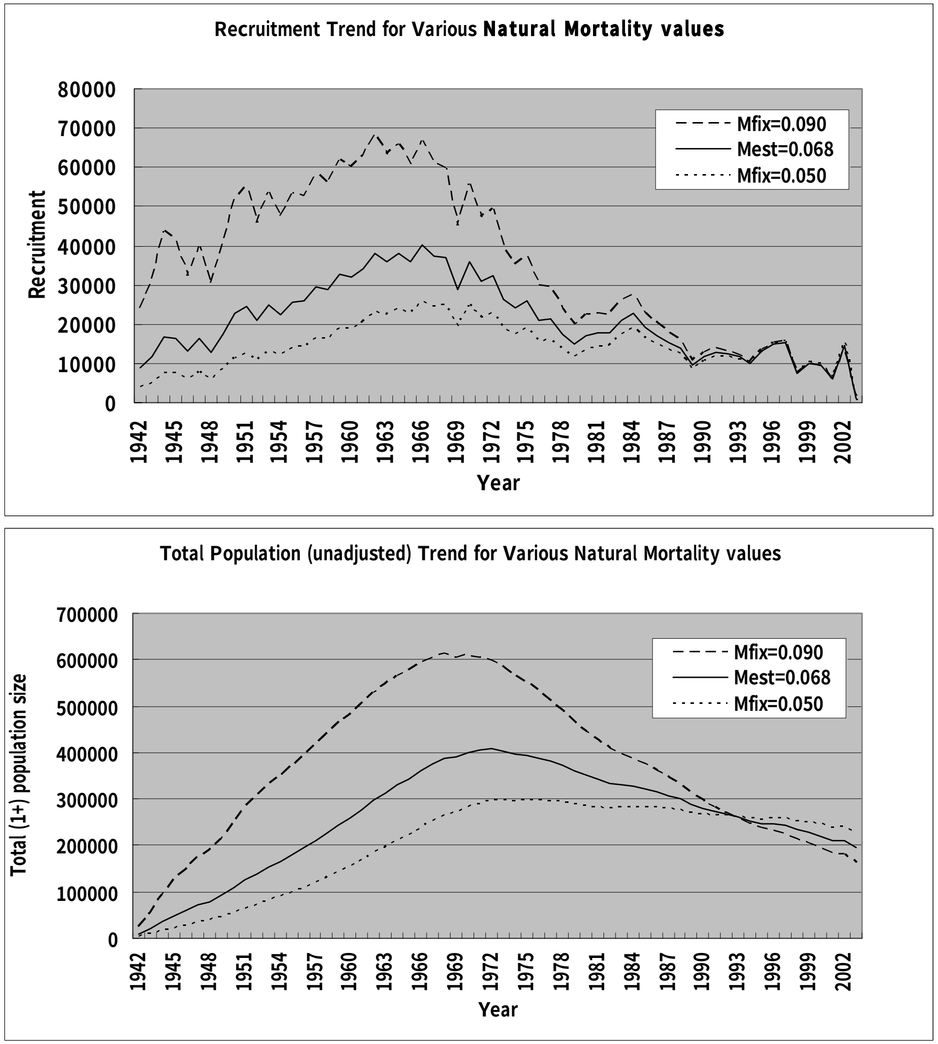

Figure 6. Estimated recruitment ($N_{y,1}$) and total population size trajectories for different choices of natural mortality M for the Areas IV+V combined assessment.

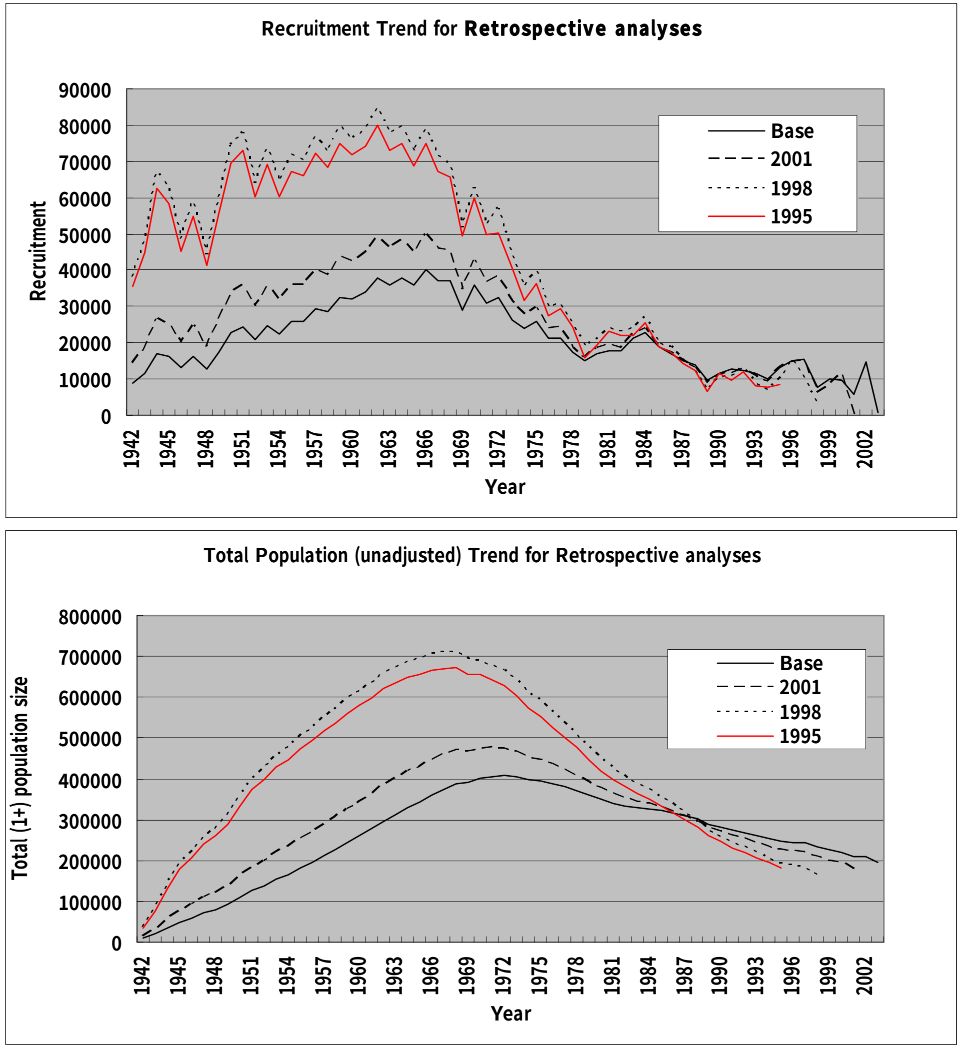

Figure 7. Estimated recruitment ($N_{y,1}$) and total population size trajectories for retrospective assessments for 2, 5 and 8 years earlier for the Areas IV+V combined assessment.

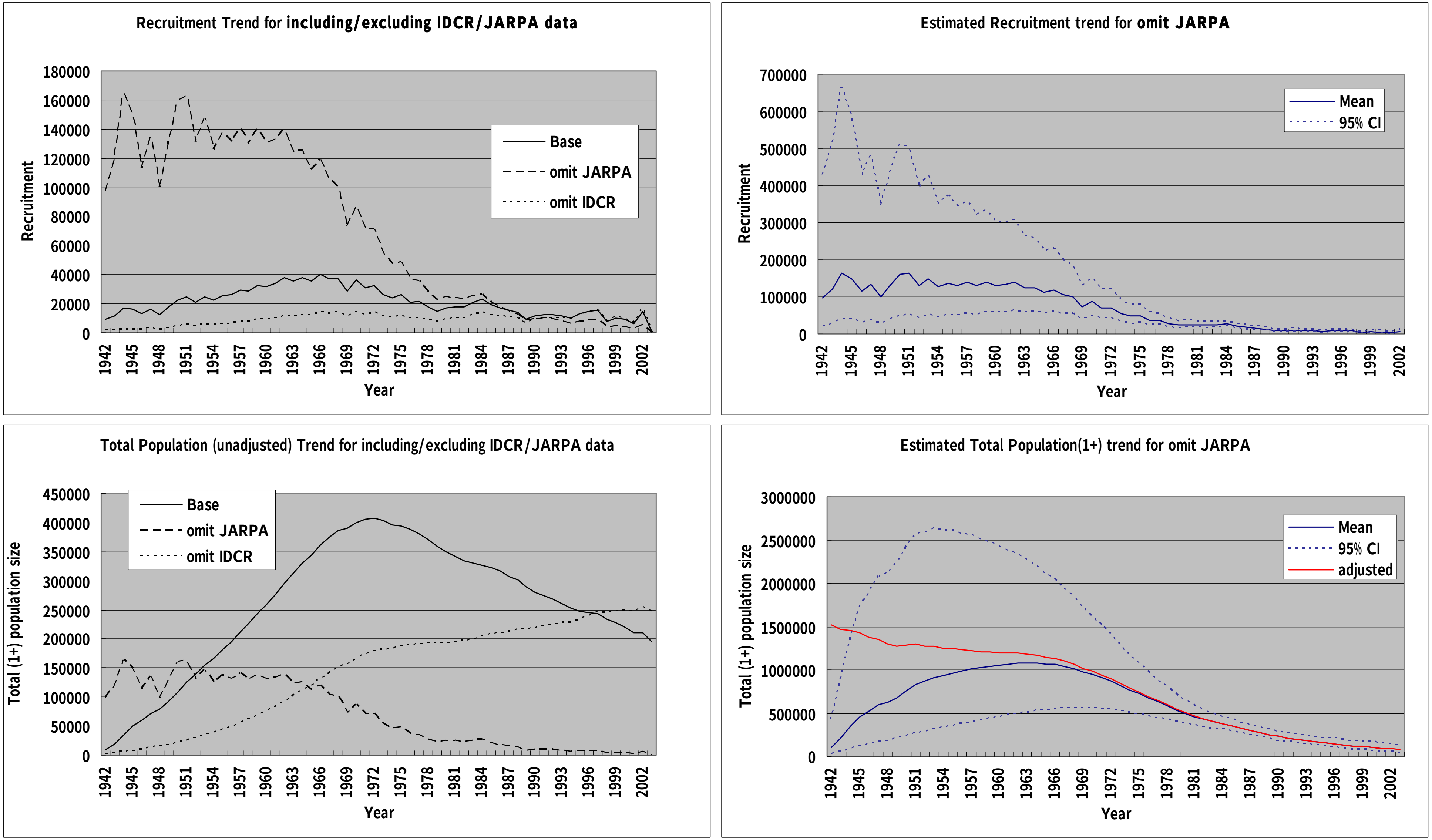

Figure 8. Estimated recruitment ($N_{y,1}$) and total population size trajectories for cases omitting either the JARPA or the IDCR/SOWER estimates of abundance for the Areas IV+V combined assessment. Confidence intervals for the former case are shown in the righthand plots.

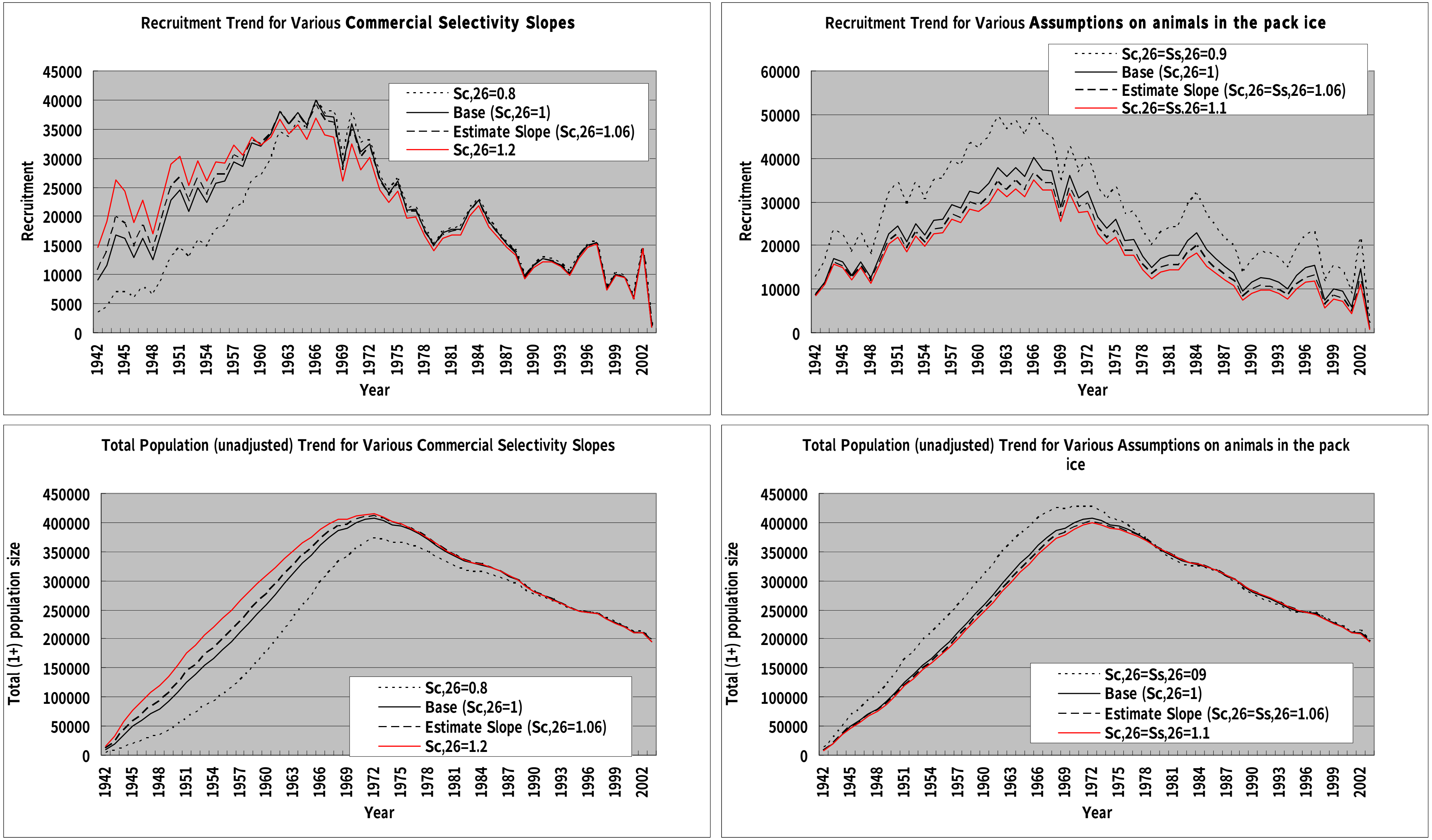

Figure 9. Estimated recruitment ($N_{y,1}$) and total population size trajectories for different choices for the commercial selectivity slope for the Areas IV+V combined assessments.

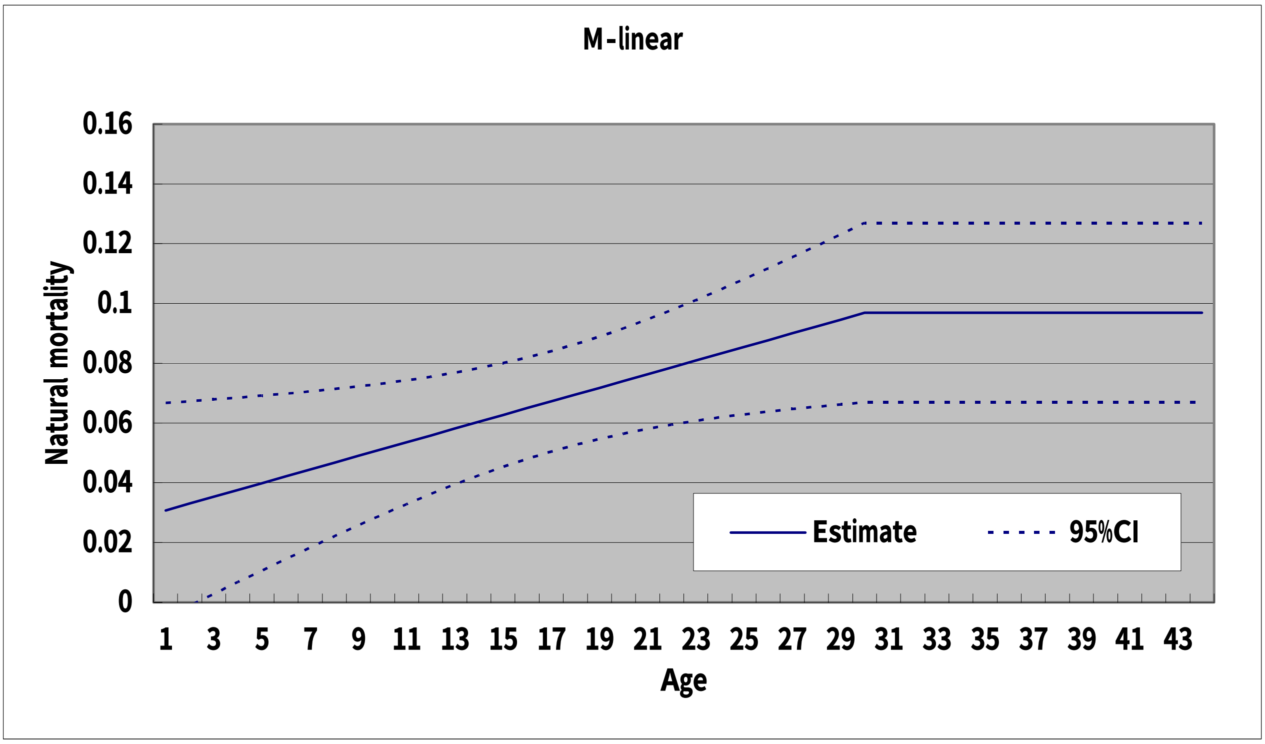

Figure 10. Confidence intervals for M_a when a linear trend over a=1 to 30 is admitted for the Areas IV+V combined assessment.

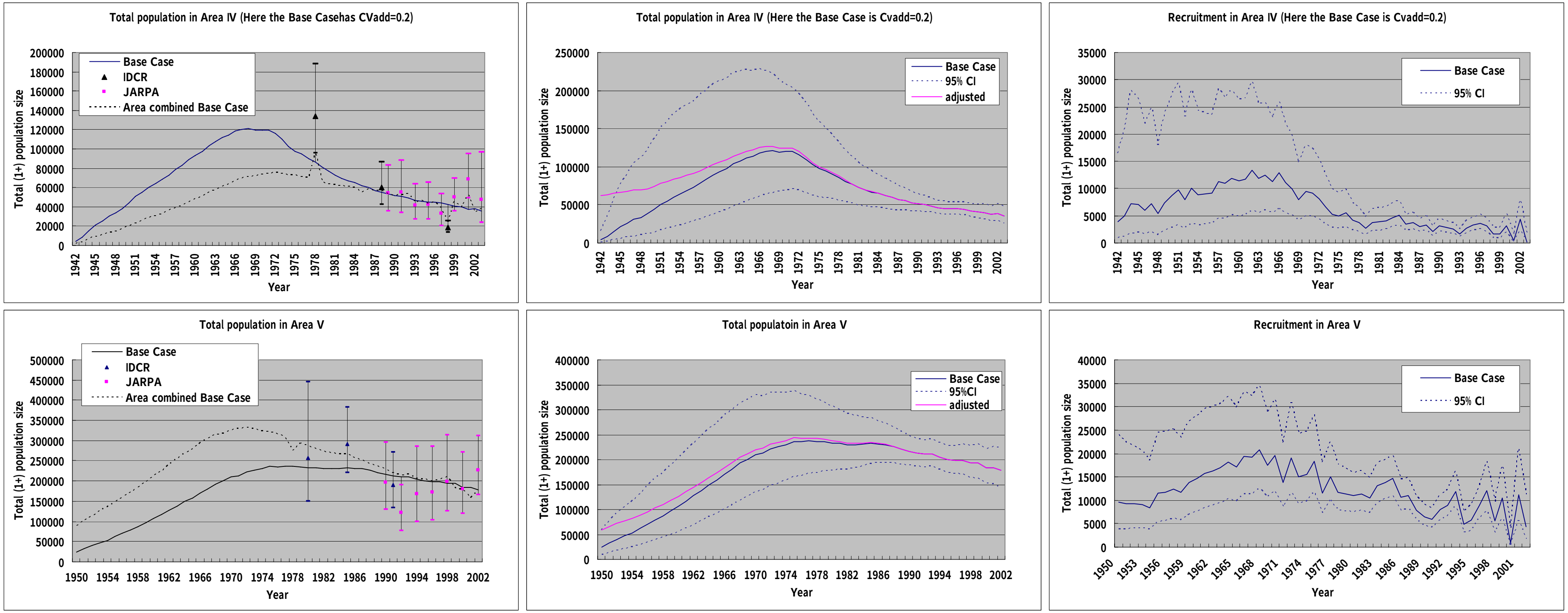

Figure 11. Base Case (see text) estimates of total population size and recruitment, together in Hessian-based estimates of 95% CI's, for separate Area IV and Area V assessments.